THE RESPONSE OF STRUCTURES TO EARTHQUAKE LOADING

A thesis presented for the degree
of Doctorate of Philosophy in Civil Engineering,
to the University of Canterbury, Christchurch,
New Zealand.

by

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1968.

ABSTRACT

This thesis considers the theoretical determination of the elasto-plastic response of structures to earthquake loading. The particular structures studied are a railway bridge and four multi-storey buildings: one with a tall flexible steel frame, two others with open reinforced concrete frames and the other a reinforced concrete spandrel beam frame. Calculations were made to determine the magnitudes of the plastic deformations which occur in these New Zealand structures under a strong motion earthquake, the N-S component of that recorded at El Centro in 1940.

The development of computer programs to calculate the elastic and elasto-plastic response of multi-mass structures to recorded earthquakes and the numerical integration methods on which they are based are outlined. Efficient static analysis programs which may be used to calculate the individual member actions in a multi-storey frame, caused by the earthquake induced lateral forces are also developed.

It was found that the lateral deflections assuming elasto-plastic behaviour were of the same order as those calculated assuming yielding did not occur. The plastic action tended to reduce the top storey deflection and increase the lower storey deflections.

The member ductilities required by these frames were of reasonable magnitude and could probably be provided with adequate detailing, but it is clear that typical structures must be

capable of providing member ductilities greater than four.

This research indicated that it is possible to design multi-storey frames so that the columns remain elastic while the beams yield and absorb energy plastically without forming a collapse mechanism.

ACKNOWLEDGEMENTS

The writer wishes to thank Professor H. J. Hopkins and Professor F. M. Henderson of the Civil Engineering Department, University of Canterbury, for their help and encouragement throughout this project. He would like to record his appreciation of the help given him by members of staff, post-graduate students and technicians of this department. He would like to thank the staff of the University of Canterbury's Computer Centre for their help with card punching and other problems.

He is particularly indebted to Mr. R. Shepherd, Reader in Civil Engineering, for his advice, assistance and encouragement as supervisor of this research work.

Mr. H. McC. Dickson, Lecturer in Civil Engineering at Monash University, Victoria, gave considerable assistance in running some of the computer programs on machines in Australia.

The writer would also like to thank Mr. R. I. Skinner of the Engineering Seismology Section of the Physics and Engineering Laboratory, D.S.I.R. Wellington, for his interest and advice; and Messrs J. P. Hollings and D. Bruce-Smith, consulting engineers, Wellington, and Mr. O. A. Glogau, Chief Structural Engineer, Ministry of Works, Wellington, for their interest in this project and for making available plans and details of actual buildings on which the structures in this thesis were modelled.

Finally he would like to gratefully acknowledge the financial assistance given him by the University Grants Committee in the form of a Post-Graduate Scholarship.

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LIST OF SYMBOLS

Symbol		Defined on Page
A	The area of member	26
A_b & A_t	Vectors of actions applied to a structural element	17
A_r	A sub-matrix of stiffness coefficients	19
$\{A\}$	A vector used in numerical integration	78
$\bar{A}$	The effective shear area	21
$\{a\}$	A vector used in numerical integration	56
B	A vector used in the solution of frame equilibrium equations	19
$\{B\}$	A vector used in numerical integration	77
$\{b\}$	A vector used in numerical integration	56
C_r	A matrix of stiffness coefficients	17,18,31
$[C]$	A Damping Matrix	54
c	A stiffness coefficient	23,35
c_r	A matrix of stiffness coefficients	30
$\bar{C}_r$	The fraction of critical damping	59
C_g & C_t	Coefficients used to specify a damping matrix	60
D	Diameter of a circle	178
D_r	A vector of frame deformations	18
d	A stiffness coefficient	23
E	Young's Modulus	15
E_r	A matrix of stiffness coefficients	30
e	A matrix of stiffness coefficients	30

F	A Force	178
$[F]$	A matrix used in numerical integration	56
$[F]_{LAT}$	The lateral flexibility matrix of a multi-mass system	28
F_s	A matrix of stiffness coefficients	31
f	A Force	15
G	The Shear modulus	21
H	The length of a member between centre lines	24
I	The Moment of Inertia of a member	15
J	The Polar Moment of Inertia	178
K	A matrix of stiffness coefficients	18,31
$[K]_{LAT}$	The lateral stiffness matrix of a multi-mass system	29
$[K^*]$	A matrix used in numerical integration	78
k	A matrix of stiffness coefficients	17
k_1	An elastic constant	178
L	The length of a member	15
M	An applied bending moment	21
m	An applied bending moment	15
$[M]$	A diagonal matrix of lumped masses	54
M_r	A vector of actions applied at floor r	31
M_y	The yield moment of a member	86
P	A vector of lateral forces applied to a frame	31
$\bar{P}$	The axial compressive force in a member	34
P_E	The Euler Buckling Load	35
p	The rotation of a plastic hinge	139

p_A & p_B	The vertical forces applied to the ends of a column member	27
p_y	A lateral force applied in direction y	35
q	The rotation of a plastic hinge	135
R	A vector of actions applied to a structural element	30
$\{\Delta R\}$	A matrix used in numerical integration	78
r	A stiffness coefficient	129
S	A vector of actions applied to a structural element	30
s	A stiffness factor	35
T	A period of vibration	87
Δt	A small increment of time	55
U	A vector of actions applied to a structural element	30
$[W^2]$	A diagonal matrix of the angular frequencies squared	61
W_r	The angular frequency of mode r	61
$\{X\}$	A vector of lateral displacements	53
$\{\dot{X}\}$	A vector of lateral velocities	53
$\{\ddot{X}\}$	A vector of lateral accelerations	53
$\ddot{X}_g$	The acceleration of the ground	53
x	A lateral coordinate	35
y	A lateral coordinate	35
y_A & y_B	The vertical movements of the ends of a column member AB	27

y_1	A vector of lateral deformations	30
Δy	A vector of incremental deformations	38
δy	A deflection	35
Z	A matrix of stiffness coefficients	30,32
α	A parameter based on axial load	35
α_1	An angle	181
β	The shear factor	22
δ	A deflection	15
$\left.\begin{matrix}\delta_b \\ \delta_t\end{matrix}\right\}$	Vectors of the deformations of a structural element	17
$\{\delta\}$	A vector of lateral displacements	183
Δ	An imposed lateral displacement	25
θ	A rotation	15
θ_r	A vector of deformations applied at floor r	31
ϕ	An instability function	36
$[\phi]$	A matrix of the normalised displacement ratios of all the modes	59
$\{\phi\}$	A matrix of the normalised displacement ratios of a mode	59
ϕ_y	The elastic limit rotation of a member	86
μ	The member ductility ratio	
$\omega(\rho)$	An instability function	37
ω_r	The angular frequency of mode r	59
ρ	The ratio of the axial load to the Euler load	35

Brackets:

$\{\ \}$ are used to denote a column vector

$[\ \ ]$ are used to denote a square matrix

$\lceil\ \ \rfloor$ are used to denote a diagonal matrix

In some cases, matrices are not distinguished by brackets, particularly sub-matrices.

CHAPTER ONE

INTRODUCTION

1.1 GENERAL

The difficulty of predicting the true response of typical buildings to actual earthquake ground motion is matched by the importance of the problem. Although since the advent of the high speed digital computer it is possible in principle to calculate the elastic or inelastic response of a given structure, this process requires accurate assessment of the dynamic properties of the building, such as stiffness and damping, and also the adequate definition of the motion of the ground during an earthquake.

Early designers realised the complexity of the problem but were forced to ignore the dynamic nature of the response and design requirements were limited to the ability to withstand a lateral force proportional to the weight of the building. This method approaches the problem as though buildings are rigid structures and the maximum ground motion is the only criterion for the design.

The introduction of the response spectrum concept by Benioff in 1934[(1)] provided a logical method of allowing for the dynamic properties of the building. Response spectra are curves relating the response, such as displacement, velocity or acceleration, usually with damping as a parameter, to the period of vibration of a simple resonator which is subjected to a recorded earthquake disturbance.

Clearly the response of a single storey building to the earthquake motion may be estimated directly from the response spectrum. The usefulness of the response spectrum approach is greatly extended by the concept that the response of a multi-mass system may be obtained by summing the response of its normal modes because each mode may be regarded as an independent one degree of freedom system, subject to certain restrictions on the damping present.

One difficulty with this approach is that the response spectrum gives the maximum response of each mode and in general these maxima do not occur at the same time. The arithmetic sum of the modal maxima is an upper limit to the maximum response of the multi-mass system but for many structures the root mean square of the modal responses provides a close approximation[2][3].

The response spectra - normal mode approach clearly distinguishes the characteristics of the structure from those of the earthquake and so the analysis is divided into two parts.

The normal mode properties of multi-storey buildings can be readily obtained using a digital computer which calculates the characteristic frequencies, displacements and shears, working from the elastic properties of the members and the mass considered to be lumped at each floor level. If a digital computer is not available, the period of the fundamental mode can be estimated if the stiffness of the building is known, and hence the response in the first mode found from the appropriate spectrum. This is usually a major proportion of the total response of the structure and for this reason some building codes[(4)(5)] now base the required design loading on the fundamental period, i.e. the code effectively contains a response spectrum.

The response spectra for various earthquakes were first calculated using electrical analogue techniques[(6)(7)] and have since been calculated using the versatile high speed digital computer[(8)].

An alternative method of determining the response of a multistorey building to a particular earthquake record is to carry out the direct integration of the equations of motion of the multimass system using a digital computer. As mentioned earlier, this procedure requires a knowledge of the dynamic stiffness of the building frame, the mass to be considered lumped at each floor level, the form and magnitude of the damping present and a reliable record of the earthquake motion.

If the damping present is small and in a certain restricted form, use can be made of the independence of the normal modes by carrying out the step-by-step integration of the equations of motion of each mode, adding the modal responses at each step and determining the maximum response of the structure.

Usually, almost all the response of the structure occurs in the first three modes and so it would only be necessary to integrate three equations in the procedure just mentioned. This avoids the uncertainty of how to combine the modal responses which occur with the response spectrum approach. It also avoids the use of an extremely small step interval which according to Newmark[9] would need to be less than one tenth of the natural period of the highest mode for the integration of the equations of motion of all the lumped masses. However, before this numerical integration of the first three modes can be carried out it is necessary to carry out a modal analysis.

The earthquake loading which can be predicted from either of the above two approaches greatly exceeds the usual design code requirements, but structures designed according to code specifications have performed satisfactorily under major earthquakes. This is generally explained by postulating that energy is absorbed by non-structural elements such as partition walls and the subsoil, and that the structural elements continue to absorb energy when strained far beyond their elastic limits.

Significant inelastic deformations will be produced in typical buildings by any reasonably severe earthquake and so research programs were directed towards the study of simple elasto-plastic systems subjected to earthquake motion.

Penzien[10] carried out an analytic investigation involving a single mass system that had an idealised elasto-plastic resistance - deformation relationship and was subjected to the E-W component of the 1940 El Centro earthquake. He used a digital computer to carry out the numerical step-by-step integration procedure. He plotted the displacement response of the system against the natural period for various combinations of ultimate strength and damping. He pointed out that buildings designed under code requirements depend on damping provided by plastic deformation to limit their dynamic response during a strong motion earthquake, and that the inelastic damping and non-structural strength are primarily responsible for the apparent ability of many existing structures to resist strong motion earthquakes.

Veletsos and Newmark[8] also presented studies of elasto-plastic single degree of freedom systems both with and without damping, subjected to strong earthquake motion. They pointed out that the maximum relative displacement of an elasto-plastic system having a ductility of four was about the same order of magnitude as the maximum deformation in the elastic system with the same initial stiffness. They also suggested the use in design of a ductility factor and stated that response and consequently the design loads for an elasto-plastic system could be derived from the corresponding quantities for the elastic system, by the use of a reduction factor. This factor is of the order of $\frac{1}{4}$ for a ductility factor of the order of 4. As they pointed out, this reduction factor brings the loading predicted for elastic systems reasonably close to that currently used in design.

Berg and Thomaides[11] also studied a damped one degree of freedom elasto-plastic system. Their results indicated that yielding does not increase the total energy input to this system and that the maximum drift may not be adversely affected provided that the yield level is maintained above a reasonable threshold.

They stated that their results suggest the possibility of using a design philosophy proposed by Housner[12] namely that structures should survive the more frequent, moderate ground motions without damage and should be capable of withstanding intense seismic shocks without total collapse occurring. This dual objective can be achieved by requiring that a structure remain elastic in its response to a moderate tremor and that it be able to consume the energy of a very intense earthquake by calling on its reserve strength beyond the yield deformation point. Berg and Thomaides pointed out that the extension of design concepts developed for single degree of freedom systems is not clear. It cannot be readily predicted where inelastic deformation will occur. It may occur in many locations in the structure or it may be concentrated in one or two locations.

Penzien[13] carried out an analytical investigation using a digital computer where he studied the behaviour of an idealised multi-storey building under the E-W component of the 1940 El Centro earthquake. He studied the effect of the parameters, natural period, yield strength and damping. He noted that the relative contribution of the higher modes to the total maximum response was considerably reduced by the introduction of a small amount of damping. The whip effect of the top storey was greatly reduced by the recommendation now included in some codes that 10% of the total design load be concentrated at the top of the structure. He showed that plastic yielding considerably reduces the response of multi-storey buildings and concluded that it is a major factor in the ability of existing structures to withstand strong seismic disturbances.

Berg[14] analysed two multi-storey frames, which were more flexible than might be expected of seismically designed frames, integrating the differential equations of motion using a method previously outlined[15]. He introduced plastic hinges in beams and columns when the yield moment was exceeded and approximated the recorded earthquake accelerogram by a piecewise linear function. He located the zero axis of the accelerogram by integrating the ground acceleration to obtain the ground velocity and displacement. Small corrections were made to the accelerogram record to give reasonable values of ground velocity and displacement. He stated that while these small corrections have a substantial effect on the computed ground motion they have a negligible effect on structural response. This is reassuring, as published results for the ground velocity and displacement of the same earthquake show wide discrepancies. Berg also stated that even though the frames became less stiff when plastic hinges are present, the amplitude of oscillation was decreased, because of the energy dissipated through plastic deformation.

The studies outlined above indicated that the maximum structural displacement amplitudes tend to be reasonably independent of the yield strength of the structures. On the basis of this observation the applicability of the elastic response spectra was extended to include any elasto-plastic structure which responds as a true one degree of freedom system. The required ductility factor is given by dividing the elastic spectral response force by the yield force of the simple system.

For more complex structures yielding may be expected to destroy the elastic mode vibration characteristics which form a basis for the mode superposition techniques. Because plastic deformations will not be distributed in a similar fashion to elastic deformations it is difficult to apply the concept of a ductility reduction factor to the design of tall multi-storey buildings.

There has not yet been any complete design method published to correspond to the design philosophy proposed by Housner[12]. It is relatively easy to estimate the behaviour and likely inertia loading under earthquake loading assuming elastic response. The frame members may then be designed by allowable stress methods using standard methods of frame analysis. This procedure assumes satisfactory behaviour under small tremors, but it is not yet possible to accurately predict the behaviour of the structure in the non-linear elasto-plastic range, and so the execution of the second part of the design philosophy, namely that structures should resist very strong seismic shocks without total collapse, is still subject to considerable doubt.

The present design techniques for frame structures, either using ultimate strength or allowable stress methods, result in some members yielding before others, under strong motion earthquakes. The questions which then should be answered are: what are the magnitudes of the plastic deformations and are the members capable of providing them; is the maximum displacement greatly altered by the plastic action; is the yielding restricted to beam members only; and does total collapse occur through mechanisms being formed?

Clough, Benuska and Wilson[16] used a digital computer program to determine the amount and distribution of plastic deformation in a typical reinforced concrete 20-storey three bay open frame building when subjected to the 1940 El Centro earthquake, N-S component. They used a step-by-step numerical integration procedure previously proposed by Wilson and Clough[17]. The structure was assumed to behave in a linear elastic manner during each time increment and the non-linear response was obtained as a sequence of successively differing systems.

Their studies indicated that even a moderate earthquake will cause incipient yielding. The lateral displacements developed in non-linear response appeared to be similar in magnitude to the elastic displacement response. For the building studied non-linear member deformations tended to concentrate in weak members, and for this reason frame design should not include weak zones which would attract a major part of the plastic deformation. They recommended designing tall buildings so that the columns respond essentially elastically and the earthquake energy is absorbed by plastic deformation in the girders.

Donald(18) presented computer methods of determining the elastic response of multi-storey buildings using the normal mode-response spectrum approach. Donald's methods were based on using the University of Canterbury's IBM 1620 computer to assemble and invert the frame stiffness matrix giving the frame flexibility matrix, from which the terms of the lateral flexibility matrix can be selected. The normal mode properties were found by iterating the lateral flexibility matrix and the member actions found by using the frame flexibility matrix to find the deformations under the applied joint loading and then substituting these deformations into the member stiffness relationships. The main difficulties with this process are that the frame stiffness and flexibility matrices are of a high order for a tall frame, so that storage problems are encountered; and that the matrix inversion procedure was extremely slow requiring approximately one hour on the IBM 1620 to invert a 50 x 50 matrix.

The writer's investigation has been aimed at analysing the behaviour of various typical New Zealand structures, using methods similar to those developed by Clough, Benuska and Wilson,(14) to give the non-linear response to strong earthquake motion and, in particular, the required member ductilities. The difficulties and danger of extending the results of the analysis of one type of elasto-plastic system to another type make it worthwhile considering the behaviour of multi-mass structures other than the frame already considered.

1.2 CONTENTS

The improvements in static analysis methods developed by the writer are described in Chapter 2. They decreased the required computer time to perform a given analysis by a factor of about 6, and doubled the size of frame which the IBM 1620 could handle.

In Chapter 3 the determination of the elastic response of multi-mass systems to earthquake records by the numerical integration of the equations of motion of the masses or the modal equations of motion is described. The method of defining the damping matrix for a frame is outlined.

The determination of the elasto-plastic response of a multi-storey building to earthquake loading using a digital computer, which considers bending deformations only, is described in Chapter 4.

In Chapter 5 the elastic and elasto-plastic analyses of the dynamic behaviour of a tall steel frame building and a small reinforced concrete building are considered.

The computer program developed in Chapter 4 is extended to include the effects of joint size and shear deformation, and the dynamic analyses of two tall reinforced concrete frame buildings are described in Chapter 6.

The elastic and elasto-plastic analyses of a large multi-span railway bridge vibrating under earthquake loading, and the development of computer programs to carry out these analyses are given in Chapter 7.

The major part of the numerical integration of the multi-mass equations was carried out in Australia using CDC 3200 and 3600 model computers. Latterly some of this work has been carried out on the University of Canterbury's IBM 360/44.

Chapter 8 is devoted to discussion of the work of this thesis and to conclusions.

CHAPTER TWO

STATIC ANALYSIS OF MULTI-STOREY FRAMES

2.1 DEVELOPMENT OF TRI-DIAGONAL TECHNIQUES

In order to overcome the problems of carrying out the static analysis of tall multi-storey buildings on a relatively slow digital computer, the IBM model 1620, with limited storage, it was decided to write more efficient computer programs than those developed by Donald[18]. These new programs enabled much larger frames to be analysed and considerably reduced the computer time required.

The development of a computer program which determines the member actions of a regular multi-storey frame under any system of joint loading is described in this section. By regular it is understood that there are no set backs and no missing members.

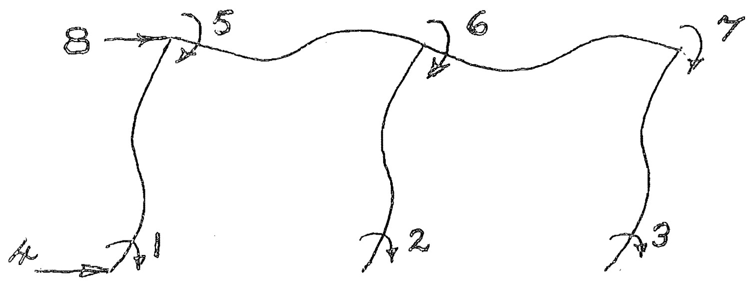
The stiffness matrices of multi-storey frames consist of a sparse array of elements which follow a regular pattern, if the deformations are referenced in a systematic manner. If the zero elements of the stiffness matrix are ignored in a computer program a considerable amount of storage may be saved in the machine. This means that much larger buildings may be analysed with a given computer and this technique was found to be particularly useful with the University of Canterbury's IBM 1620 computer.

The other advantage of ignoring the zero elements of the stiffness matrix is that the amount of computation is considerably reduced; this means that the time of computation is much less and that the magnitude of the truncation errors will be reduced.

The essential feature of the method is the division of the structure into discrete elements in such a way that the non-zero terms of the frame stiffness matrix may be partitioned into sub-matrices arranged in a tri-diagonal matrix form.

One way of dividing the structure is to regard the members of each storey level as forming an element.

For example, consider the third floor element of a two-bay multi-storey frame:



We may assemble the stiffness matrix for the storey element by adding the appropriate stiffness coefficients for the component members. If bending deformations only are considered, the equilibrium equations for a column member may be written as follows:

$$\begin{Bmatrix} m_i \\ f_j \\ m_k \\ f_l \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 4 & \frac{6}{L} & 2 & -\frac{6}{L} \\ \frac{6}{L} & \frac{12}{L^2} & \frac{6}{L} & -\frac{12}{L^2} \\ 2 & \frac{6}{L} & 4 & -\frac{6}{L} \\ -\frac{6}{L} & \frac{12}{L^2} & \frac{6}{L} & +\frac{12}{L^2} \end{bmatrix} \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \\ \delta_l \end{Bmatrix}$$

where m denotes an action which is a bending moment,

f " " " " " " force,

θ " a deformation which is a rotation,

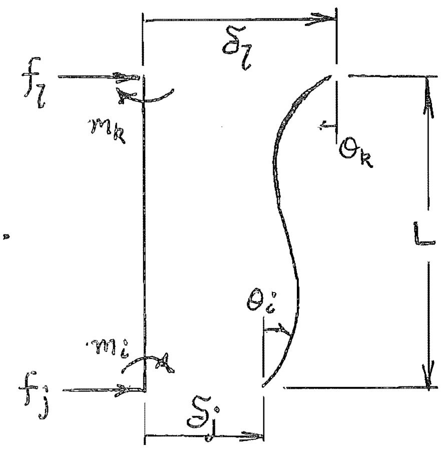
δ " " " " " " deflection,

E denotes Young's modulus for the material of the member,

I denotes the second moment of area of the member,

L denotes the length of the member.

Subscripts are used to distinguish individual actions and deformations. Deformations which correspond to a given action are denoted with the same subscript. The position of the actions and deformations are shown in the diagram:

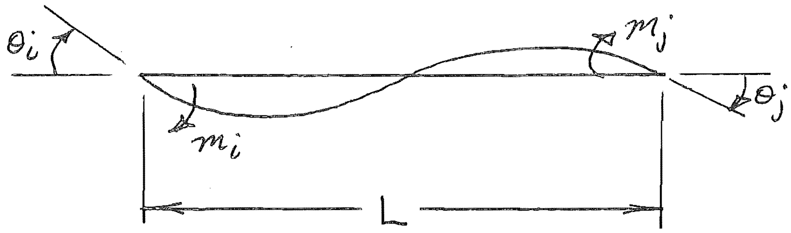


EI constant

The equilibrium equations for a beam may be written:

$$\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix}$$

using similar notation, for the beam shown in the diagram;



The storey stiffness matrix is assembled by adding all of the stiffness coefficients of the component members into the appropriate positions. The deformations of the storey element are numbered in a systematic manner as shown for the two bay element above. These numbers are also used as subscripts i, j, k and l for the appropriate actions and deformations of the component members. The row in the storey stiffness matrix to which the member stiffness coefficient is added is given by the subscript of the associated action and the column by the subscript of the associated deformation.

If the deformations at the floor level which forms the top of the element are separated from the deformations at the floor level which forms the bottom of the element, we may partition the matrix equilibrium equations for the storey element as follows:

$$\begin{Bmatrix} A_b \\ A_t \end{Bmatrix} = \begin{bmatrix} k_b & C \\ C^T & k_t \end{bmatrix} \begin{Bmatrix} \delta_b \\ \delta_t \end{Bmatrix}$$

where A_b is a column vector of actions applied at the bottom of the element,

A_t is the vector of actions at the top,

δ_b and δ_t are the vectors of deformations which correspond to A_b and A_t respectively,

k_b is a sub-matrix of stiffness coefficients relating the actions and deformations at the bottom of the element,

k_t is the sub-matrix relating actions and deformations at the top of the element,

C is the sub-matrix of stiffness coefficients relating the actions at the top of the element to the deformations at the bottom.

The frame stiffness matrix is assembled by combining the storey element stiffness matrices. Deformations imposed at any particular floor level of a multi-storey frame induce actions only at that floor level and the floor levels immediately above and below. Hence if the frame deformations are partitioned so that the deformations at a particular floor level form a sub-matrix, and the frame actions and frame stiffness matrix similarly partitioned then the only non-zero sub-matrices of stiffness coefficients, corresponding to the actions at a particular level, are those which relate to the deformations at the floor level above, the floor itself and the floor below. Thus any row of the stiffness matrix consists of only three non-zero sub-matrices.

The equilibrium equations for the frame may then be written in the following form:

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{n-1} \\ A_n \end{Bmatrix} = \begin{bmatrix} K_1 & C_1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ C_1^T & K_2 & C_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & C_2^T & K_3 & C_3 & \cdot & \cdot & \cdot \\ & & & \ddots & & & \\ & & & & C_{n-2}^T & K_{n-1} & C_{n-1} \\ & & & & \cdot & C_{n-1}^T & K_n \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_{n-1} \\ D_n \end{Bmatrix}$$

where A_r denotes a column vector of the actions applied at the rth floor level,

D_r denotes a column vector of the corresponding deformations,

K_r denotes a sub-matrix of stiffness coefficients which relate the actions at floor level r to the deformations at level r,

C_r denotes a sub-matrix of stiffness coefficients which relate the actions at floor level r to the deformations at level r+1.

To assemble the frame stiffness matrix the sub-matrices from the component storey elements are added to the appropriate position. This is done by adding the sub-matrix k_t, from the storey element containing the beams of floor r, to the sub-matrix k_b from the element above to form sub-matrix K_r. Sub-matrix C_r is identical to sub-matrix C from the element above floor r.

The frame equilibrium equations may now be solved by elimination taking advantage of the tri-diagonal form. Firstly the stiffness matrix is reduced to the lower diagonal form by matrix operations:

$$\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_n \end{Bmatrix} = \begin{bmatrix} I & \cdot & \cdot & & \\ A_1 & I & \cdot & & \\ \cdot & A_2 & I & & \\ & & \ddots & \ddots & \\ & & & A_{n-1} & I \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{Bmatrix}$$

It is desirable to carry out the elimination operations as the sub-matrices are set up and in this way it is only necessary to store the lower diagonal of sub-matrices i.e. approximately one third of the non-zero stiffness coefficients.

The solution for vector D_1 is found directly from the first row, i.e. B_1, and then a recursive procedure is followed where the vector found from the previous row is substituted in the next row which is solved to give a new vector.

In the computer program "T.D.E." for which a block diagram (Figure 2.1) and listing (List 1) follow, the lower diagonal of sub-matrices is stored as a three dimensional array A, the third subscript being used to indicate the position of the sub-matrix on the diagonal. The storey element sub-matrix k_t is added to the matrix B, the sub-matrix k_b is stored as matrix C, the sub-matrix C^T is stored in the appropriate position of A.

The program is limited to the analysis of a frame up to 20 storeys high by 4 bays wide, without using symmetry. The program was dimensioned to fit into the storage of the IBM 1620 so that larger frames could be handled on other machines such as the IBM 360/44.

The member actions are found by a second program "T.D.E.M.A." (List 2) which accepts the deformations determined by the first program and calculates the member actions by determining the member stiffness matrix from the member properties and multiplying by the member deformations.

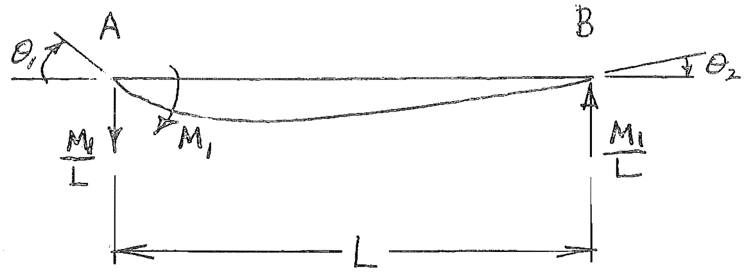
2.2. INCLUSION OF SHEAR AND AXIAL DEFORMATION, AND JOINT SIZE EFFECTS

It has been shown by Donald[18] that for some frames the effects of shear and axial deformation and joint size are significant, particularly frames with short deep beams or in the pierced wall type of frame.

In the following section member stiffness matrices are derived taking account of these effects.

1. Shear deformation

Consider a simple beam AB with moment M_1 applied at end A



Then using Mohr's dummy unit load method, by considering the effect of the deformation of a small element and integrating over the length of the beam:

$$\theta_1 = \frac{L}{3EI} M_1 + \frac{1}{\bar{A}G} \frac{M_1}{L}$$

$$\theta_2 = -\frac{L}{6EI} M_1 + \frac{1}{\bar{A}G} \frac{M_1}{L}$$

where θ_1 and θ_2 are the rotations at ends 1 and 2 respectively, clockwise positive,

E and I are as previously defined,

$\bar{A}$ is the effective shear area,

G is the shear modulus.

Similarly if a moment M_2 is applied at end B

$$\theta_2 = \frac{L}{3EI} M_2 + \frac{1}{\bar{A}G} \frac{M_2}{L}$$

$$\theta_1 = -\frac{L}{6EI} M_2 + \frac{1}{\bar{A}G} \frac{M_2}{L}$$

Hence the equilibrium equations for the beam written in flexibility form are

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{bmatrix} \left(\frac{L}{3EI} + \frac{1}{\bar{A}GL}\right) & \left(-\frac{L}{6EI} + \frac{1}{\bar{A}GL}\right) \\ \left(-\frac{L}{6EI} + \frac{1}{\bar{A}GL}\right) & \left(\frac{L}{3EI} + \frac{1}{\bar{A}GL}\right) \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}$$

Let $\beta = \frac{6EI}{L^2\bar{A}G}$

then

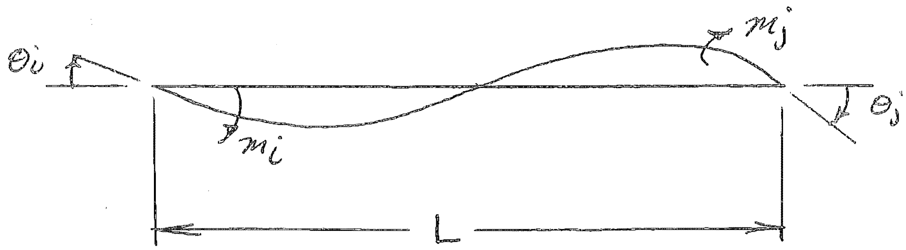
$$\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \frac{L}{6EI} \begin{bmatrix} (2+\beta) & (-1+\beta) \\ (-1+\beta) & (2+\beta) \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}$$

By inverting the flexibility matrix, the stiffness matrix (k) is found to be

$$(k) = \frac{2EI}{L} \frac{1}{1+2\beta} \begin{bmatrix} (2+\beta) & (1-\beta) \\ (1-\beta) & (2+\beta) \end{bmatrix}$$

Hence the stiffness matrices for the beam and column elements may be written as follows, making allowance for the sway of the column chord

Beam element

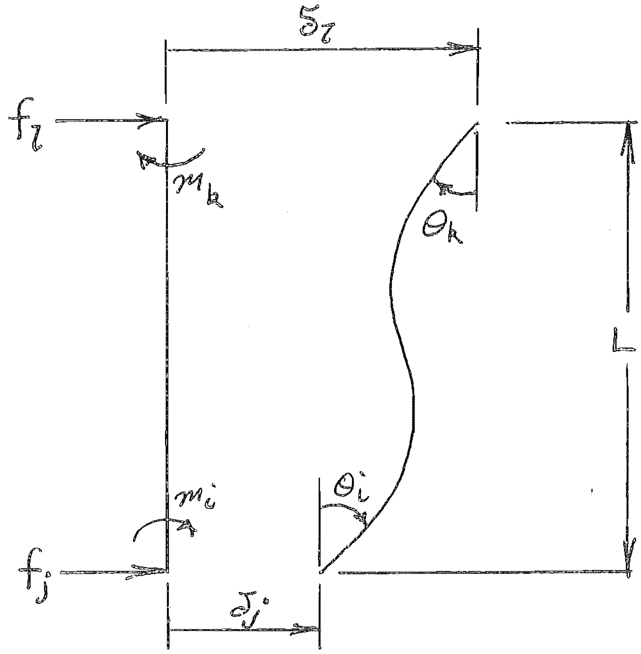


$$\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} = \begin{bmatrix} c & d \\ d & c \end{bmatrix} \begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix}$$

where

$$c = \frac{2EI}{L} \; \frac{2 + \beta}{1 + 2\beta} \qquad d = \frac{2EI}{L} \; \frac{1 - \beta}{1 + 2\beta}$$

Column element



$$\begin{Bmatrix} m_i \\ f_j \\ m_k \\ f_l \end{Bmatrix} = \begin{bmatrix} c & \frac{c+d}{L} & d & -\frac{c+d}{L} \\ \frac{c+d}{L} & \frac{2(c+d)}{L^2} & \frac{c+d}{L} & -\frac{2(c+d)}{L^2} \\ d & \frac{c+d}{L} & c & -\frac{c+d}{L} \\ -\frac{c+d}{L} & -\frac{2(c+d)}{L^2} & -\frac{c+d}{L} & +\frac{2(c+d)}{L^2} \end{bmatrix} \cdot \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \\ \delta_l \end{Bmatrix}$$

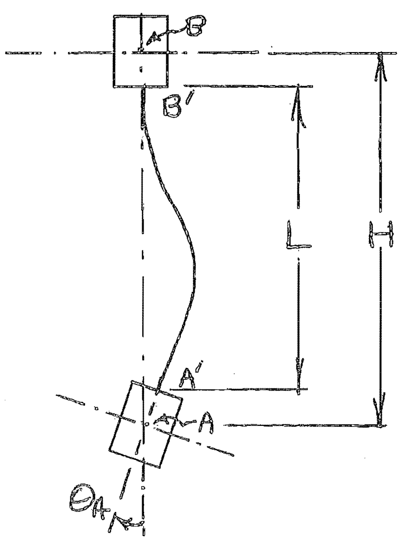
Joint size

The joint is considered to be a completely rigid block. The stiffness matrix for a member with rigid end blocks is found by calculating the actions at the centre line intersection for a unit deformation applied at the centre line intersection.

The member stiffness matrices above are used to calculate the actions at A' and B' and then the actions at A and B are determined by considering the equilibrium of the blocks at A and B.

Consider a column member with the following deformations imposed:

a) Angle θ_A at A



Then lateral displacement at A' $= \frac{H-L}{2}\theta_A$

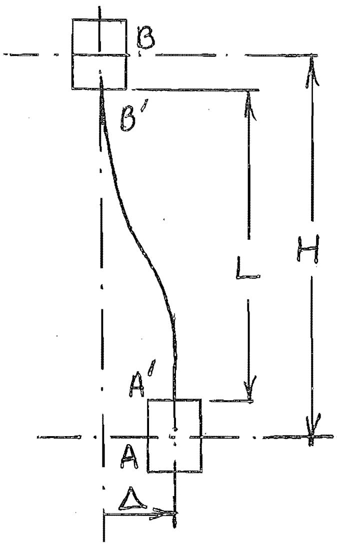
and $m_A = (c + \frac{c+d}{2} \; \frac{H^2-L^2}{L^2}) \cdot \theta_A$

$m_B = (d + \frac{c+d}{2} \; \frac{H^2-L^2}{L^2}) \cdot \theta_A$

$f_A = -f_B = \frac{c+d}{L^2} \; H \; \theta_A$

Similar relationships may be derived for an angle θ imposed at B.

b) Lateral displacement Δ at A



Then $m_A = \frac{c+d}{L} \cdot \frac{H}{L} \Delta$

$$f_A = \frac{2(c+d)}{L^2} \Delta$$

$$m_B = \frac{c+d}{L} \cdot \frac{H}{L} \Delta$$

Hence the member stiffness matrix for a column member with rigid end blocks may be derived

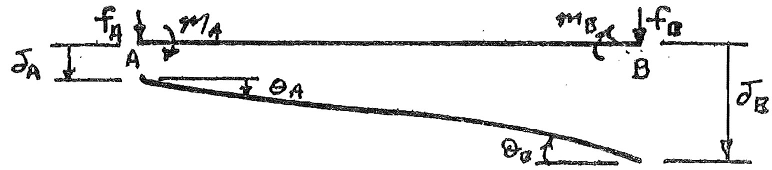
$$\begin{Bmatrix} m_A \\ f_A \\ m_B \\ f_B \end{Bmatrix} = \begin{bmatrix} \left(c + \frac{c+d}{2} \cdot \frac{H^2-L^2}{L^2}\right) & \left(\frac{c+d}{L^2} H\right) & \left(d + \frac{c+d}{2} \cdot \frac{H^2-L^2}{L^2}\right) & \left(-\frac{c+d}{L^2} H\right) \\ & 2\,\frac{c+d}{L^2} & \frac{c+d}{L^2} H & -\frac{2(c+d)}{L^2} \\ & & \left(c + \frac{c+d}{2} \cdot \frac{H^2-L^2}{L^2}\right) & -\frac{c+d}{L^2} H \\ \text{symmetrical} & & & +\frac{2(c+d)}{L^2} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \delta_A \\ \theta_B \\ \delta_B \end{Bmatrix}$$

The member stiffness matrix for a beam member may be derived in a similar way and contains the coefficients which relate to the rotations only.

Axial Deformation

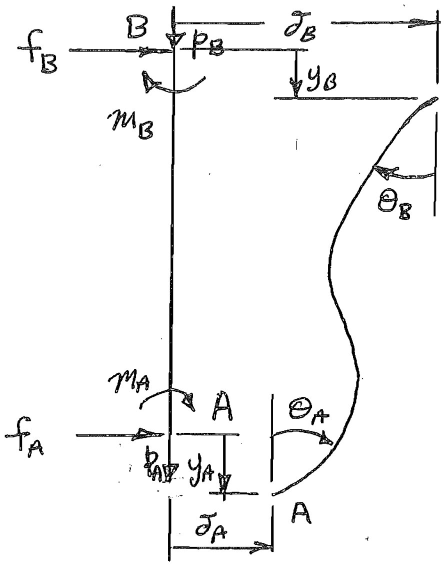
Only the linear effects of axial loads are considered; the axial loads not being considered to contribute to the bending moment distribution i.e. the axial deformation in a column of length L, cross-sectional area A, Young's modulus E and compressive force $\bar{P}$ is assumed to be given by $\frac{\bar{P}L}{EA}$.

When axial deformation is considered the vertical deflection of the ends of a beam must be taken into account. The stiffness coefficients derived for the sway of a column chord are directly applicable. The equilibrium equations for a beam element may be written:



$$\begin{Bmatrix} m_A \\ m_B \\ f_A \\ f_B \end{Bmatrix} = \begin{bmatrix} \left(c + \frac{c+d}{2}\,\frac{H^2-L^2}{L^2}\right) & \left(d + \frac{c+d}{2}\,\frac{H^2-L^2}{L^2}\right) & \left(\frac{c+d}{L^2}H\right) & \left(-\frac{c+d}{L^2}H\right) \\ & \left(c + \frac{c+d}{2}\,\frac{H^2-L^2}{L^2}\right) & \left(\frac{c+d}{L^2}H\right) & \left(-\frac{c+d}{L^2}H\right) \\ & & \left(\frac{2(c+d)}{L^2}\right) & \left(-\frac{2(c+d)}{L^2}\right) \\ \text{symmetrical} & & & \left(\frac{2(c+d)}{L^2}\right) \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \\ \delta_A \\ \delta_B \end{Bmatrix}$$

and the equilibrium equations for a column element:



$$
\begin{Bmatrix} m_A \\ f_A \\ p_A \\ m_B \\ f_B \\ p_B \end{Bmatrix}
=
\begin{bmatrix}
\left(c + \frac{c+d}{2}\,\frac{H^2-L^2}{L^2}\right) & \left(\frac{c+d}{L^2}H\right) & 0 & \left(d + \frac{c+d}{2}\,\frac{H^2-L^2}{L^2}\right) & \left(-\frac{c+d}{L^2}H\right) & 0 \\
 & \left(\frac{2(c+d)}{L^2}\right) & 0 & \left(\frac{c+d}{L^2}H\right) & \left(-\frac{2(c+d)}{L^2}\right) & 0 \\
 & & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} \\
 & & & \left(c + \frac{c+d}{2}\,\frac{H^2-L^2}{L^2}\right) & \left(-\frac{c+d}{L^2}H\right) & 0 \\
 & & & & \frac{2(c+d)}{L^2} & 0 \\
\text{symmetrical} & & & & & \frac{EA}{L}
\end{bmatrix}
\begin{Bmatrix} \theta_A \\ \delta_A \\ y_A \\ \theta_B \\ \delta_B \\ y_B \end{Bmatrix}
$$

A computer program "MODTDE" was written incorporating the above member stiffness matrices and is shown in List 3. The stiffness matrix is set up with a tri-diagonal band of sub-matrices and solved, in the same way as detailed for the program which considered bending deformations only, for the joint deformations of the frame. The program is limited to the analysis of a frame up to 14 storeys by 5 bays if axial deformation is ignored or up to 15 storeys by 2 bays if axial deformation is considered. With lateral loading only, the number of bays may be doubled by considering symmetry. A second program "MTDEMA" (List 4) calculates the member actions, at the centre line intersections and also at the member faces, from the joint deformations and member properties.

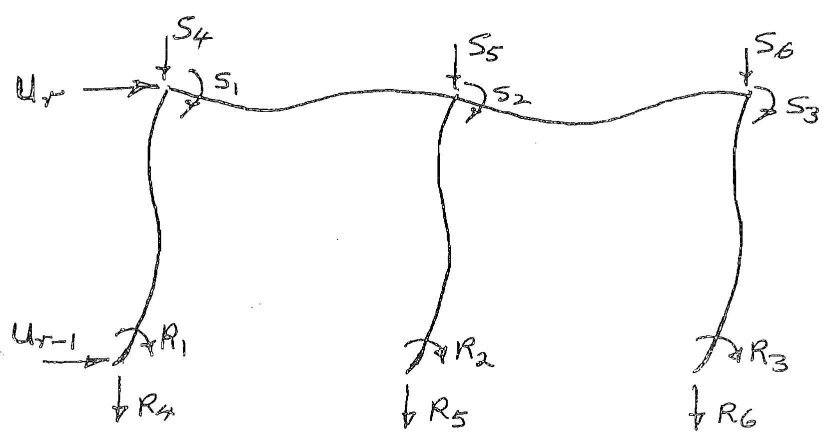
2.3 DETERMINATION OF THE LATERAL STIFFNESS MATRIX

It is useful to have a computer program which can derive the lateral flexibility matrix from the sparse frame stiffness matrix without having to invert the whole stiffness matrix and then store the whole frame flexibility matrix which is not sparsely populated.

This section describes a computer program which sets up the stiffness matrix storing only the non-zero elements in a tri-diagonal band with two off-diagonal bands of sub-matrices. The stiffness matrix is reduced by matrix operations to the lateral stiffness matrix which is inverted to give the lateral flexibility matrix, $[F]_{LAT}$. This is an efficient way of calculating $[F]_{LAT}$

with the added advantage of producing, as an intermediate step, the lateral stiffness matrix $\left[K\right]_{LAT}$. There are occasions in dynamic analysis work when it is preferable to have $\left[K\right]_{LAT}$ for a frame rather than $\left[F\right]_{LAT}$; e.g. the determination of the elastic response to an earthquake record by numerical integration, the calculation of the damping matrix, and in determining the overall flexibility of a building composed of individual frames.

The method is based on writing the frame equilibrium equations so that the rotations and axial deformations are separated from the lateral displacements. The matrix equilibrium equations for the storey element are partitioned so that the rotations at the bottom floor level, the rotations at the top floor level and the two lateral displacements are formed into three groups. If axial deformations are considered they are grouped with the adjacent rotation. For example consider the rth storey element of a two bay frame:



The equilibrium equations for this element may be written:

$$\begin{Bmatrix} R \\ S \\ \mathcal{U} \end{Bmatrix} = \begin{bmatrix} k_{r-1} & c_{r-1} & e_{r-1} \\ c^T_{r-1} & k_r & e_r \\ e^T_{r-1} & e^T_r & Z \end{bmatrix} \cdot \begin{Bmatrix} r \\ s \\ y_l \end{Bmatrix}$$

where $R = \begin{Bmatrix} R_1 \\ R_2 \\ \vdots \\ R_6 \end{Bmatrix}$ $\quad S = \begin{Bmatrix} S_1 \\ S_2 \\ \vdots \\ S_6 \end{Bmatrix}$ $\quad \mathcal{U} = \begin{Bmatrix} u_{r-1} \\ u_r \end{Bmatrix}$

r, s, and y_l are the deformations corresponding to R, S and $\mathcal{U}$ respectively.

k_{r-1}, c_{r-1}, e_{r-1}, k_r, e_r and Z are sub-matrices which form the storey element stiffness matrix.

The frame stiffness matrix is again assembled by combining the storey element stiffness matrices. The equilibrium equations for the frame may then be written as follows:

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_N \\ P \end{Bmatrix} = \begin{bmatrix} K_1 & C_1 & \circ & \circ & \cdots & E_1 \\ C^T_1 & K_2 & C_2 & \circ & \cdots & E_2 \\ \circ & C^T_2 & K_3 & C_3 & \cdots & E_3 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ \circ & \circ & \circ & \circ & K_N & E_N \\ E_1^T & E_2^T & E_3^T & E_4^T \cdots & E_N^T & F_S \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_N \\ X \end{Bmatrix}$$

where M_r is a vector of the moments and vertical forces at floor level r,

θ_r is a vector of the rotations and vertical axial deformations at floor level r,

P is a vector of the lateral forces applied to the frame,

X is a vector of the lateral deflections,

K_r denotes a sub-matrix of stiffness coefficients which relate the actions at floor r to the deformations at floor r,

C_r denotes a sub-matrix of stiffness coefficients which relate the actions at floor r to the deformations at floor r+1,

E_r denotes a sub-matrix of stiffness coefficients which relate the actions at floor r to the lateral deflections of the frame,

K_r is formed by adding the matrices k_r from the element above floor r and the element below,

E_r is similarly formed by adding the two matrices e_r,

C_r is identical to the matrix c_r,

F_S is formed by adding the matrices Z.

The vectors M_1 to M_N are null because only lateral force actions are applied to the frame. The constraints against vertical and rotational deformation may be relaxed and the relationship between lateral loads and lateral displacement found.

By matrix operations the equilibrium equations are reduced to:

$$\left\{ \begin{matrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ P \end{matrix} \right\} = \begin{bmatrix} I & C_1' & 0 & 0 & \cdots & E_1' \\ 0 & I & C_2' & 0 & \cdots & E_2' \\ 0 & 0 & I & C_3' & \cdots & E_3' \\ & & \vdots & \vdots & \vdots & \vdots \\ & & & & I & E_N' \\ & & & & & Z \end{bmatrix} \left\{ \begin{matrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_N \\ X \end{matrix} \right\}$$

The relationship between P the vector of applied lateral forces and ∂ the vector of lateral displacements is now given by:

$$\{P\} = [Z]\{X\}$$

independent of the other deformations. $[Z]$ is the lateral stiffness matrix for the frame, $[K]_{LAT}$.

There follows a listing (see List 5, p.49) of a computer program "KLAT" which determines the lateral stiffness matrix, and then inverts this to give $[F]_{LAT}$ if desired, for a regular multi-storey frame considering shear deformation, axial deformation and joint size if desired.

The stiffness matrix for each storey element is set up in turn starting at the bottom of the frame, and matrix operations are carried out to reduce the stiffness matrix to upper triangular form. The stiffness coefficients from the storey element below are added to those from the element above. The sub-matrices are set up and elimination operations carried out storey by storey and in this way only nine sub-matrices are needed to store the stiffness matrix.

Suppose we have reached the following stage in the elimination operations:

$$\begin{bmatrix} I & C_1' & \circ & \circ & \circ & \circ & \cdots & E_1' \\ \circ & I & C_2' & \circ & \circ & \circ & \cdots & E_2' \\ \circ & \circ & I & C_3' & \circ & \circ & \cdots & E_3' \\ \circ & \circ & \circ & K_4 & C_4 & \circ & \cdots & E_4 \\ \circ & \circ & \circ & C_4^T & K_5 & \circ & \cdots & E_5 \\ \circ & \circ & \circ & \circ & C_5^T & K_6 & \cdots & E_6 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ \circ & \circ & \circ & E_4^T & E_5^T & E_6^T & \cdots & F \end{bmatrix}$$

Then it is only necessary to store the matrices

$$\begin{bmatrix} K_4 & C_4 & & E_4 \\ C_4^T & K_5 & & E_5 \\ & & & \\ E_4^T & E_5^T & & F \end{bmatrix}$$

Matrices C_5^T, K_6 and C_6 have not yet been set up.

These nine sub-matrices are then stored by the program as:

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & Z \end{bmatrix}$$

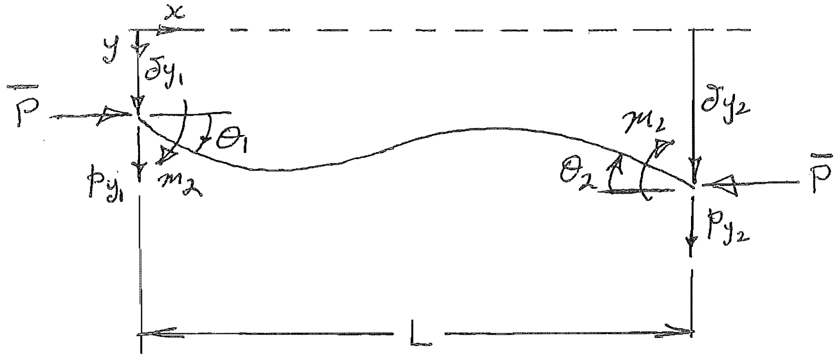
and are referred to by this notation.

2.4 INSTABILITY

So far the stiffness matrices for a uniform member in plane bending have been developed assuming the only effect of the axial component of load was to produce axial strains. These matrices must be modified if the contribution which the axial thrust makes to the bending moment distribution is considered.

Livesley(19) has derived stiffness matrices which take account of the non-linear effects of axial loads. Consider a uniform member AB and assume that the compressive force in the member, $\overline{P}$, is a known parameter. Then assuming the lateral displacement of the member is sufficiently small for the elementary theory of bending to apply, we may write:

$$EI\frac{d^2y}{dx^2} = p_{y_1}.x - \overline{P}(y - \delta y_1) - m_1$$



Integrating this differential equation and inserting the appropriate end conditions, the following modified slope - deflection equations are obtained:

$$m_1 = \frac{EI}{L^2} s(1+c)\,\delta y_1 + \frac{EI}{L} s\,\theta_1 - \frac{EI}{L^2} s(1+c)\,\delta y_2 + \frac{EI}{L} sc\,\theta_2$$

$$m_2 = \frac{EI}{L^2} s(1+c)\,\delta y_1 + \frac{EI}{L} sc\,\theta_1 - \frac{EI}{L^2} s(1+c)\,\delta y_2 + \frac{EI}{L} s\,\theta_2$$

where E and I are as previously defined

and $s = \dfrac{(1-2\alpha\cot 2\alpha)\,\alpha}{\tan\alpha - \alpha}$ where $\alpha = \dfrac{\pi}{2}\sqrt{\rho}$

and $c = \dfrac{2\alpha - \sin 2\alpha}{\sin 2\alpha - 2\alpha\cos 2\alpha}$

$$\rho = \frac{\bar{P}}{P_E}$$

$$P_E = \frac{\pi^2 EI}{L^2}$$

P_E is the Euler Buckling Load.

The end shears Py_1 and Py_2 are found by applying the condition of moment equilibrium

$$m_1 + m_2 + Py_2 L + \bar{P}(\delta y_2 - \delta y_1) = 0$$

then

$$P_{y_1} = -P_{y_2} = \left\{\frac{2EI}{L^3}\,s(1+c) - \frac{\overline{P}}{L}\right\}\delta y_1 + \frac{EI}{L^2}\,s(1+c)\,\theta_1$$

$$-\left\{\frac{2EI}{L^3}\,s(1+c) - \frac{\overline{P}}{L}\right\}\delta y_2 + \frac{EI}{L^2}\,s(1+c)\,\theta_2$$

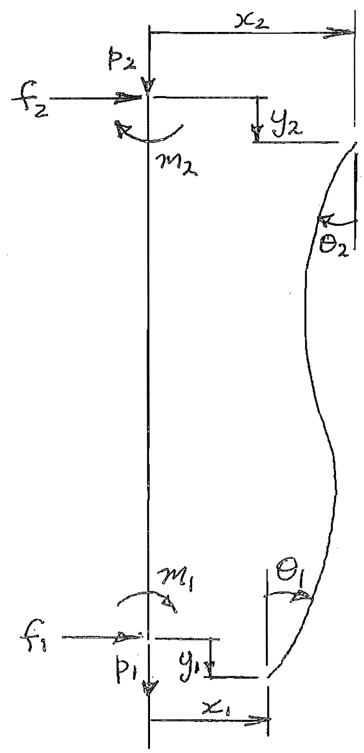
Because the axial force $\overline{P}$ has been regarded as a known parameter of the member, the above equilibrium equations are linear in the deformation components δy_1, θ_1 etc.

By introducing the functions Φ_1, Φ_2, Φ_3, Φ_4

$$\Phi_1 = \frac{s(1+c)}{6} - \frac{\overline{P}L^2}{12EI} \qquad \Phi_2 = \frac{s(1+c)}{6}$$

$$\Phi_3 = \frac{s}{4} \qquad \Phi_4 = \frac{sc}{2}$$

The equilibrium equations for a member may be written in a more familiar form:



$$\begin{Bmatrix} m_1 \\ f_1 \\ p_1 \\ m_2 \\ f_2 \\ p_2 \end{Bmatrix} = \begin{bmatrix} \frac{4EI}{L}\Phi_3 & \frac{6EI}{L^2}\Phi_2 & 0 & \frac{2EI}{L}\Phi_4 & -\frac{6EI}{L^2}\Phi_2 & 0 \\ & \frac{12EI}{L^3}\Phi_1 & 0 & \frac{6EI}{L^2}\Phi_2 & -\frac{12EI}{L^3}\Phi_1 & 0 \\ & & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} \\ & & & \frac{4EI}{L}\Phi_3 & -\frac{6EI}{L^2}\Phi_2 & 0 \\ & & & & \frac{12EI}{L^3}\Phi_1 & 0 \\ \text{symmetrical} & & & & & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ x_1 \\ y_1 \\ \theta_2 \\ x_2 \\ y_2 \end{Bmatrix}$$

Comparing these equations with those on page 14, it is clear that the Φ functions are merely factors for the stiffness coefficients for a member without end thrust.

The values of the Φ functions may be calculated for a given value of axial thrust using the following equations:

$$s = \frac{\pi^2 \rho}{4(1-\omega(\rho))} + \omega(\rho)$$

$$c = \frac{1}{s}\left\{\frac{\pi^2 \rho}{4(1-\omega(\rho))} - \omega(\rho)\right\}$$

where $\omega(\rho) = \frac{\pi}{2}\sqrt{\rho}\cot\left(\frac{\pi}{2}\sqrt{\rho}\right) = \alpha\cot\alpha$

This function $\omega(\rho)$ possesses singularities and the normal

Taylor series expansion does not converge very well. Livesley[19] has suggested calculating the function as the sum of a power series and a rational function:

$$\omega(\rho) = \frac{64 - 60\rho + 5\rho^2}{64 - 20\rho + \rho^2} - \sum_{1}^{7} \frac{a_n \rho^n}{2^{3n}}$$

where $a_1 = 1.5797363$

$a_2 = 0.15858587$

$a_3 = 0.02748899$

$a_4 = 0.00547540$

$a_5 = 0.00115281$

$a_6 = 0.00024908$

$a_7 = 0.00005452$

The stiffness coefficients for the members of a frame may be calculated if we know the axial thrusts in the members. Fortunately the axial thrusts in the members of multi-storey frames may be estimated fairly accurately by regarding the beams as being simply supported. The equilibrium equations for the frame may then be set up using the estimated axial forces and these equations solved for the joint deformations under the external joint actions. The internal member actions may be found by substituting the appropriate joint deformations into the member stiffness relationship. The axial thrusts in the members will be obtained as part of the computed internal forces and moments; and the analysis is consistent only if the computed axial forces agree with those assumed.

A computer program "INSTAB" has been written (List 6) which carries out the above static analysis using axial forces which are read as part of the input data and are estimated from the applied vertical joint loads. If the calculated axial forces do not agree with those assumed, then the program assumes the axial forces are equal to those just calculated as part of the internal member actions. A further analysis is then carried out and this iteration procedure is repeated until the computed axial forces agree with those assumed in calculating the member stiffness coefficients. The program was run on the IBM 360/44 computer and has been dimensioned so that up to a 15 storey 4 bay frame can be analysed.

The static analysis is carried out in exactly the same manner as the programs detailed previously "T.D.E" and "MODTDE", the only differences being in the subroutines which calculate the member stiffness coefficients, and in the introduction of an iteration procedure.

The critical loads for three frames were determined by investigating the behaviour, under a constant disturbance, as the vertical loading was increased. A small lateral load was applied to the top floor; this procedure being designed to excite the side-sway buckling mode. The member properties are shown in the elevations in Figure 2.2 and the reduction in lateral stiffness as the vertical loading is increased is shown in Figure 2.3. The critical load is given by the intercept with the x - axis. The frames were considered free to buckle only in the plane of the loading.

PROGRAM "T.D.E."
BLOCK DIAGRAM
FOR LIST 1

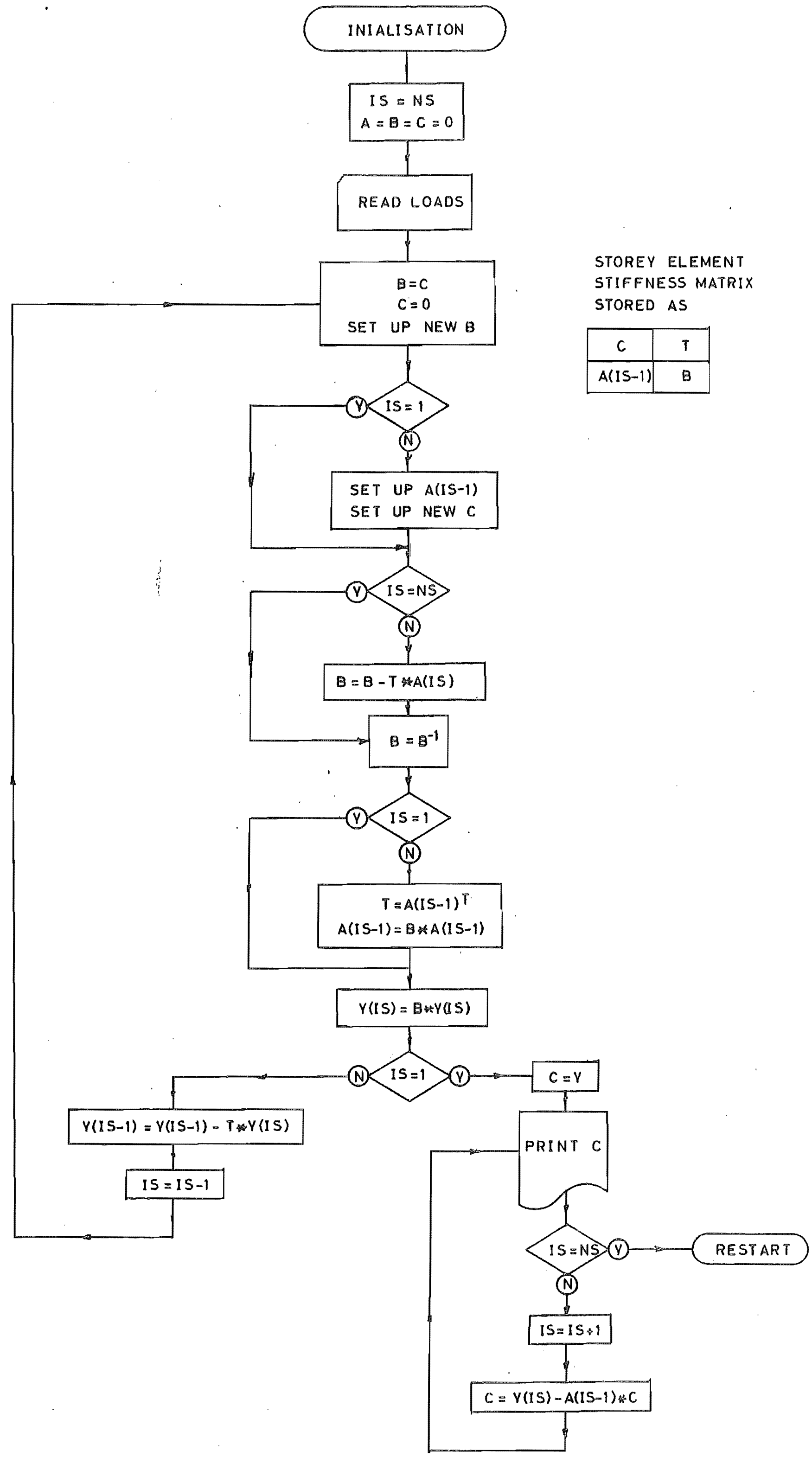
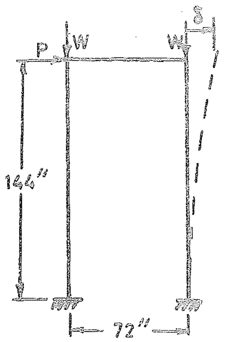


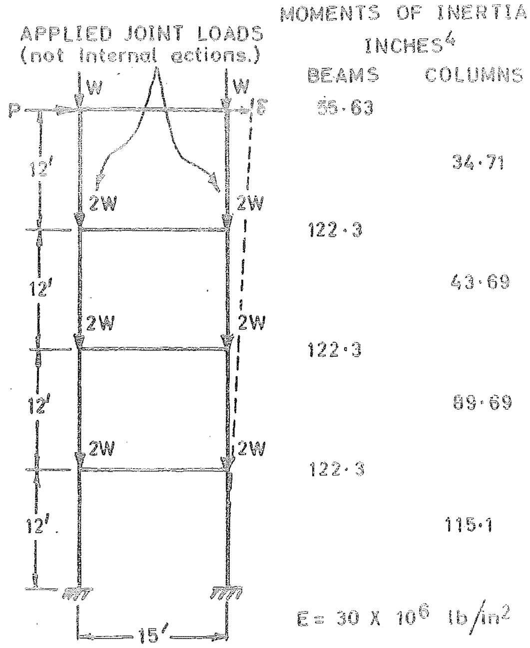
FIGURE 2·1

FRAME 2.

FRAME 1.



EI = 39·6 X 10^6 lb/in^2



FRAME 3.

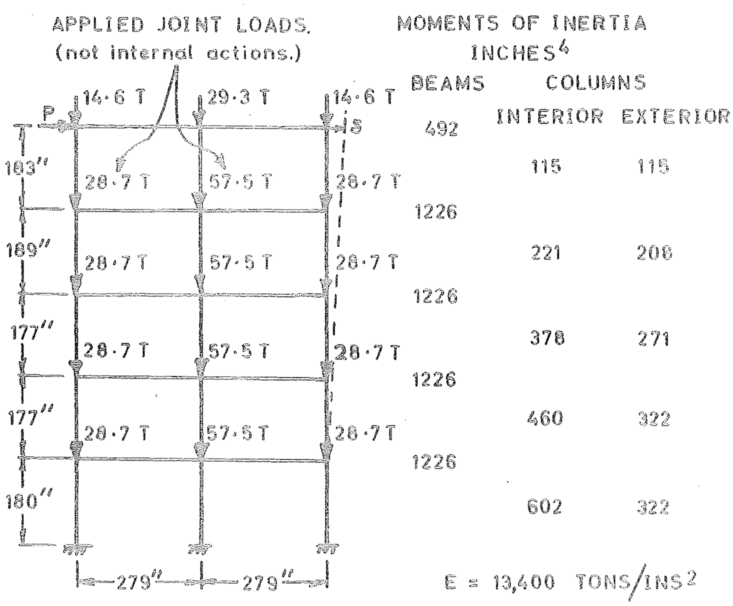


FIGURE 2·2

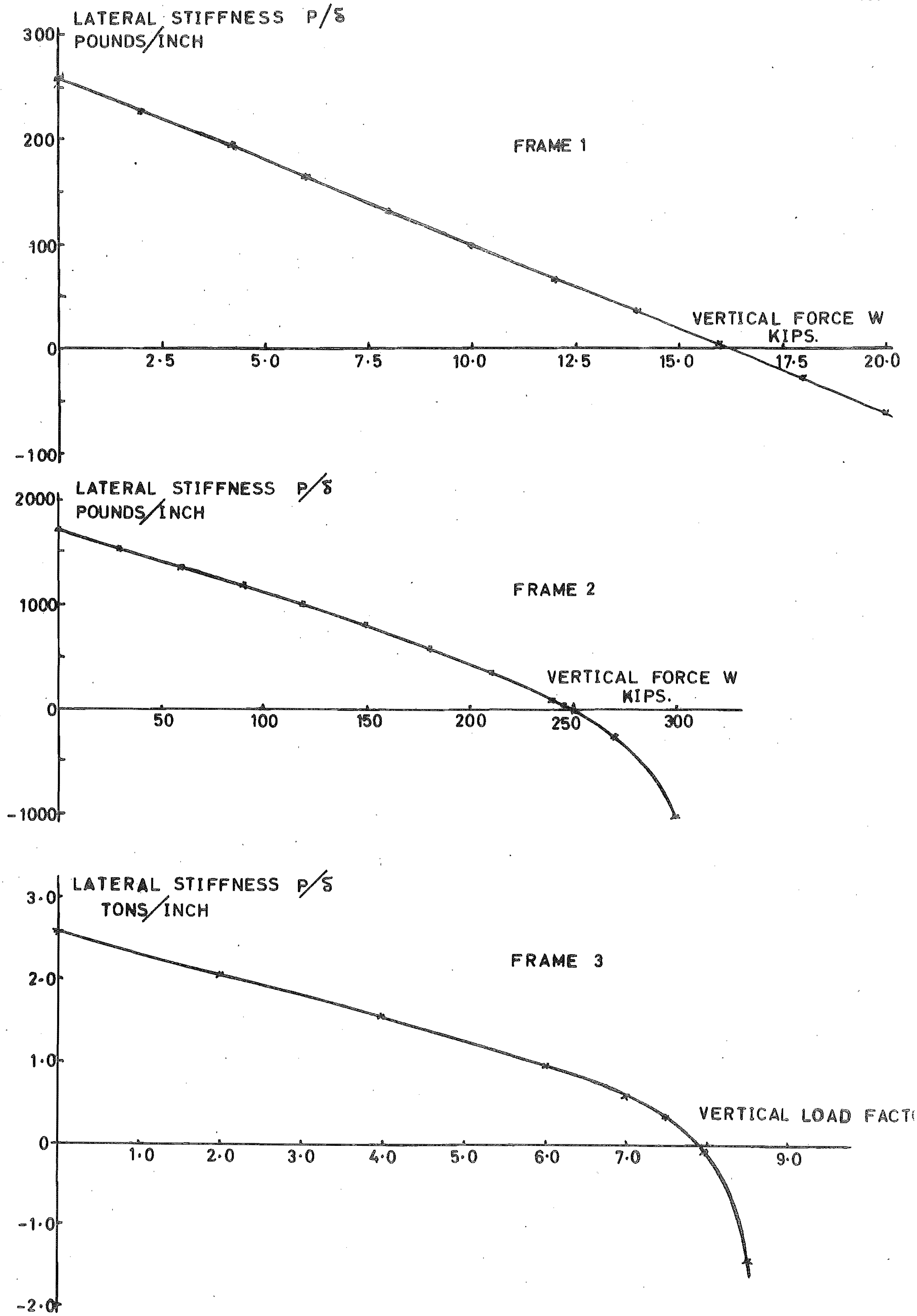

LATERAL STIFFNESS P/δ
POUNDS/INCH
300
200
100
0
-100
FRAME 1
VERTICAL FORCE W
KIPS.
2·5
5·0
7·5
10·0
12·5
15·0
17·5
20·0
LATERAL STIFFNESS P/δ
POUNDS/INCH
2000
1000
0
-1000
FRAME 2
VERTICAL FORCE W
KIPS.
50
100
150
200
250
300
LATERAL STIFFNESS P/δ
TONS/INCH
3·0
2·0
1·0
0
-1·0
-2·0
FRAME 3
VERTICAL LOAD FACT
1·0
2·0
3·0
4·0
5·0
6·0
7·0
9·0

FIGURE 2·3

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C      PROGRAM ''T.D.E.''   LIST 1                        PAGE 1

C      LANGUAGE - FORTRAN II FOR IBM 1620.
C
C      THIS PROGRAM CALCULATES THE JOINT DEFORMATIONS OF A REGULAR
C      MULTI-STOREY FRAME UP TO 20 STOREYS BY 4 BAYS WITH 5 LOADING
C      CASES, CONSIDERING BENDING DEFORMATIONS ONLY, USING
C      TRI-DIAGONAL ELIMINATION.
C
C      IL=1 DETM. OF ALL FRAME DEFORMATIONS.
C      IL=2 DETM. OF LATERAL DISPLACEMENTS ONLY.
C      IM=1 READ LATERAL FORCES ONLY, ZERO MOMENT ASSUMED
C      IM=2 READ FORCES AND MOMENTS
       DIMENSIONA(6,6,19),B(6,6),C(6,6),Y(6,20,5),TEMP(6,6),TEMPV(6,5)
C      NS=NO OF STOREYS, NB=NO OF BAYS, IHB=1 FOR A HALF BAY,
C      NL=NO OF LOADS, E=YOUNGS MODULUS.
    42 READ29,NS,NB,IHB,NL,E,IL,IM
    29 FORMAT(4I3,E10.4,2I3)
       NB2=NB&2
       NS1=NS-1
       DO3I=1,NB2
       DO3J=1,NB2
       B(I,J)=0.
       C(I,J)=0.
       DO3K=1,NS1
     3 A(I,J,K)=0.
       NB1=NB&1
       IS=NS
C      READ APPLIED LOADS
       GO TO (51,52),IM
    51 DO53I=1,NB1
       DO53J=1,NS
       DO53L=1,NL
    53 Y(I,J,L)=0.
       READ33,((Y(NB2,J,L),J=1,NS),L=1,NL)
       GO TO 2
    52 READ33,(((Y(I,J,L),I=1,NB2),J=1,NS),L=1,NL)
    33 FORMAT(8E10.4)
       NL=NS
     2 DO50I=1,NB2
       DO50J=1,NB2
       B(I,J)=C(I,J)
    50 C(I,J)=0.
       IS1=IS-1
C      READ BEAM DATA
       DO13I=1,NB
       READ32,P,Q
    32 FORMAT(2E10.4)
       P=P*E/Q
       Q=6.0/Q
    10 B(I,I)=B(I,I)&4.0*P
       B(I,I&1)=B(I,I&1)&2.0*P
       B(I&1,I)=B(I,I&1)
    13 B(I&1,I&1)=B(I&1,I&1)&4.0*P
       IF(IHB-1)14,15,15
C      READ HALF-BEAM DATA, FULL LENGTH, ACTUAL I
    15 READ32,P,Q
       B(NB1,NB1)=B(NB1,NB1)&6.0*P*E/Q
    14 J=NB2
C      READ COLUMN DATA
       DO12I=1,NB1
       READ32,P,Q
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C      PROGRAM ''T.D.E.''   LIST 1                        PAGE 2
       P=P*E/Q
       Q=6.0/Q
       B(I,I)=B(I,I)&4.0*P
       B(I,J)=B(I,J)-Q*P
       B(J,I)=B(I,J)
       B(J,J)=B(J,J)&Q*Q*.33333333*P
       IF(IS-1)12,12,11
    11 C(I,I)=C(I,I)&4.0*P
       C(I,J)=C(I,J)&Q*P
       C(J,I)=C(I,J)
       C(J,J)=C(J,J)&Q*Q*.33333333*P
       A(I,I,IS1)=A(I,I,IS1)&2.0*P
       A(I,J,IS1)=A(I,J,IS1)&Q*P
       A(J,I,IS1)=A(J,I,IS1)-Q*P
       A(J,J,IS1)=A(J,J,IS1)-Q*Q*.33333333*P
    12 CONTINUE
C      REDUCE (K) TO LOWER DIAGONAL FORM BY ELIMINATION
       IF(IS-NS)16,17,17
    16 DO18L=1,NB2
       DO18I=1,NB2
       DO18J=1,NB2
    18 B(I,L)=B(I,L)-TEMP(I,J)*A(J,L,IS)
    17 DO20I=1,NB2
       T=B(I,I)
       B(I,I)=1.0
       DO21J=1,NB2
    21 B(I,J)=B(I,J)/T
       DO20L=1,NB2
       IF(L-I)22,20,22
    22 T=B(L,I)
       B(L,I)=0.
       DO23J=1,NB2
    23 B(L,J)=B(L,J)-T*B(I,J)
    20 CONTINUE
       IF(IS-1)24,24,25
    25 DO26I=1,NB2
       DO26J=1,NB2
    26 TEMP(I,J)=A(J,I,IS1)
       DO27I=1,NB2
       DO27J=1,NB2
       A(I,J,IS1)=0.
       DO27L=1,NB2
    27 A(I,J,IS1)=A(I,J,IS1)&B(I,L)*TEMP(J,L)
    24 DO4L=1,NL
       DO19I=1,NB2
       TEMPV(I,L)=0.
       DO19J=1,NB2
    19 TEMPV(I,L)=TEMPV(I,L)&B(I,J)*Y(J,IS,L)
       DO4I=1,NB2
     4 Y(I,IS,L)=TEMPV(I,L)
       IF(IS-1)1,1,5
     5 DO28L=1,NL
       DO28I=1,NB2
       DO28J=1,NB2
    28 Y(I,IS1,L)=Y(I,IS1,L)-TEMP(I,J)*Y(J,IS,L)
       IS=IS-1
       GO TO 2
C      CARRY OUT BACK SUBSTITUTION
     1 DO40L=1,NL
       DO40I=1,NB2
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C     PROGRAM ''T.D.E.''   LIST 1
   40 C(I,L)=Y(I,1,L)
      IS=1
   48 GO TO (45,46),IL
C     PUNCH FRAME DEFORMATIONS FOR A FLOOR.
   45 DO60L=1,NL
   60 PUNCH34,(C(I,L),I=1,NB2)
      GO TO 47
   34 FORMAT(5E14.8)
   46 PUNCH34,(C(NB2,L),L=1,NL)
   47 IF(IS-NS)41,42,42
   41 IS=IS&1
      IS1=IS-1
      DO43L=1,NL
      DO43I=1,NB2
      TEMPV(I,L)=0.
      DO43J=1,NB2
   43 TEMPV(I,L)=TEMPV(I,L)&A(I,J,IS1)*C(J,L)
      DO44L=1,NL
      DO44I=1,NB2
   44 C(I,L)=Y(I,IS,L)-TEMPV(I,L)
      GO TO 48
      END
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C     PROGRAM ''T.D.E.M.A.''          LIST 2.
C     LANGUAGE - FORTRAN II FOR IBM 1620.
C
C     THIS PROGRAM CALCULATES THE MEMBER ACTIONS OF A REGULAR
C     MULTI-STOREY FRAME, FROM THE FRAME DEFORMATIONS FOUND BY
C     THE PROGRAM 'T.D.E.'
C
      DIMENSION SD(132,20)
    1 READ29,NS,NB,IHB,NL,E
   29 FORMAT(4I3,E10.4)
      NB2=NB&2
      NB1=NB&1
      DO2IS=1,NS
      K1=(IS-1)*NB2&1
      K2=K1&NB2-1
C     READ DEFORMATIONS AS PUNCHED BY PROGRAM 'T.D.E.'
      DO2IY=1,NL
      DO2I=K1,K2,5
    2 READ34,SD(I,IY),SD(I&1,IY),SD(I&2,IY),SD(I&3,IY),SD(I&4,IY)
   34 FORMAT(5E14.8)
      IS=NS
C     READ BEAM DATA
    3 IU=(IS-1)*NB2
      DO5I2=1,NB
      READ32,P,Q
   32 FORMAT(8E10.4)
      P=P*E/Q
      I=IU&I2
      K=I&1
      DO5IL=1,NL
      R1=(4.0*SD(I,IL)&2.0*SD(K,IL))*P
      R2=(2.0*SD(I,IL)&4.0*SD(K,IL))*P
      R3=(R1&R2)/Q
    5 PUNCH33,R1,R3,R2
      IF(IHB-1)6,7,7
    7 READ32,P,Q
      P=P*E/Q
      I=IU&NB1
      DO8IL=1,NL
      R1=6.0*P*SD(I,IL)
      R3=(R1&R1)/Q
    8 PUNCH33,R1,R3
    6 IU1=(IS-2)*NB2
C     READ COLUMN DATA
      DO9I2=1,NB1
      READ32,P,Q
      P=P*E/Q
      Q1=6.0/Q
      I=IU1&I2
      J=IU
      K=IU&I2
      L=IU&NB2
      DO9IY=1,NL
      R1=(2.0*SD(K,IY)-Q1*SD(L,IY))*P
      R2=(4.0*SD(K,IY)-Q1*SD(L,IY))*P
      R3=(R1&R2)/Q
      IF(IU)9,9,12
   12 R1=R1&(4.0*SD(I,IY)&Q1*SD(J,IY))*P
      R2=R2&(2.0*SD(I,IY)&Q1*SD(J,IY))*P
      R3=(R1&R2)/Q
    9 PUNCH33,R1,R3,R2
   33 FORMAT(E10.4,4X,E10.4,4X,E10.4)
      IS=IS-1
      IF(IS)1,1,3
      END
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C     PROGRAM ''MODTDE''    LIST 3.                         PAGE 1
C     LANGUAGE - FORTRAN II FOR IBM 1620.
C
C     THIS PROGRAM CALCULATES THE JOINT DEFORMATIONS OF A REGULAR
C     MULTI-STOREY FRAME UP TO 14 STOREYS BY 5 BAYS IGNORING
C     AXIAL DEFORMATIONS AND BY 2 BAYS WITH AXIAL DEFORMATIONS,
C     AND UP TO 2 LOADING CASES. SHEAR DEFORMATION AND
C     JOINT SIZE ARE CONSIDERED.
C
C     IAD=1 IGNORES AXIAL DEFORMATION.
C     IAD=2 CONSIDERS AXIAL DEFORMATION.
C     IM=1 READ LATERAL FORCES ONLY, OTHER ACTIONS ASSUMED ZERO.
C     IM=2 READ ALL ACTIONS.
C     READ MEMBER DATA TOP STOREY TO BOTTOM
C     IL=1 ALL FRAME DEFORMATIONS PUNCHED
C     IL=2 LATERAL DISPLACEMENTS ONLY PUNCHED.
      DIMENSION A(7,7,13),B(7,7),C(7,7),Y(7,14,2),
     1TEMP(7,7),TEMPV(7,2)
      COMMON R,P,G,F,H,Q,AT,SE,SR,E
C     NS=NO OF STOREYS,NB=NO OF BAYS,NL=NO OF LOADS,
C     IHB=1 OFR A HALF-BAY,SR=RATIO OF G/E (.45 FOR R.C.)
C     SE=EFFECTIVE SHEAR AREA FACTOR (1.2 FOR RECT.)
C     E=YOUNGS MODULUS.
   42 READ29,NS,NB,NL,IAD,IHB,SR,SE,E,IL,IM
   29 FORMAT(5I3,F4.2,F4.1,E10.4,2I3)
      NB2=IAD*NB&1&IAD
      NS1=NS-1
      NB1=NB&1
      DO3I=1,NB2
      DO3J=1,NB2
      B(I,J)=0.
      C(I,J)=0.
      DO3K=1,NS1
    3 A(I,J,K)=0.
      IS=NS
C     READ APPLIED LOADS
      GO TO (54,55),IM
   54 DO56I=1,NB2
      DO56J=1,NS
      DO56K=1,NL
   56 Y(I,J,K)=0.
      READ33,((Y(NB2,J,K),J=1,NS),K=1,NL)
      GO TO 2
   55 READ33,(((Y(J,K,I),J=1,NB2),K=1,NS),I=1,NL)
   33 FORMAT(8E10.4)
    2 DO50I=1,NB2
      DO50J=1,NB2
      B(I,J)=C(I,J)
   50 C(I,J)=0.
      IS1=IS-1
C     READ BEAM DATA LEFT TO RIGHT
      IF(NB)61,61,62
   62 DO80I=1,NB
      CALL STIFFS
      I1=I&1
      B(I,I)=B(I,I)&R
      B(I,I1)=B(I,I1)&P
      B(I1,I)=B(I,I1)
      B(I1,I1)=B(I1,I1)&R
      GO TO (80,81),IAD
   81 K=NB1&I
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C     PROGRAM ''MODTDE''    LIST 3.                         PAGE 2
      K1=K&1
      B(I,K)=B(I,K)&F
      B(K,I)=B(I,K)
      B(I,K1)=B(I,K1)-F
      B(K1,I)=B(I,K1)
      B(K,K)=B(K,K)&G
      B(K,I1)=B(K,I1)&F
      B(I1,K)=B(K,I1)
      B(K,K1)=B(K,K1)-G
      B(K1,K)=B(K,K1)
      B(I1,K1)=B(I1,K1)-F
      B(K1,I1)=B(I1,K1)
      B(K1,K1)=B(K1,K1)&G
   80 CONTINUE
   61 IF(IHB-1)14,15,15
C     READ HALF BEAM DATA, FULL LENGTH, ACTUAL I
   15 CALL STIFFS
      B(NB1,NB1)=B(NB1,NB1)&F*H
      GO TO (14,52),IAD
   52 J=NB1*2
      B(NB1,J)=B(NB1,J)&G*H
      B(J,NB1)=B(NB1,J)
      B(J,J)=B(J,J)&2.0*G
C     READ COLUMN DATA LEFT TO RIGHT
   14 J=NB2
      DO12I=1,NB1
      CALL STIFFS
      B(I,I)=B(I,I)&R
      B(I,J)=B(I,J)-F
      B(J,I)=B(I,J)
      B(J,J)=B(J,J)&G
      GO TO (6,7),IAD
    7 K=NB1&I
      B(K,K)=B(K,K)&AT
    6 IF(IS-1)12,12,11
   11 C(I,I)=C(I,I)&R
      C(I,J)=C(I,J)&F
      C(J,I)=C(I,J)
      C(J,J)=C(J,J)&G
      GO TO (8,9),IAD
    9 C(K,K)=C(K,K)&AT
    8 A(I,I,IS1)=A(I,I,IS1)&P
      A(I,J,IS1)=A(I,J,IS1)&F
      A(J,I,IS1)=A(J,I,IS1)-F
      A(J,J,IS1)=A(J,J,IS1)-G
      GO TO (12,10),IAD
   10 A(K,K,IS1)=A(K,K,IS1)-AT
   12 CONTINUE
C     REDUCE (K) TO LOWER DIAGONAL FORM BY ELIMINATION
      IF(IS-NS)16,17,17
   16 DO18L=1,NB2
      DO18I=1,NB2
      DO18J=1,NB2
   18 B(I,L)=B(I,L)-TEMP(I,J)*A(J,L,IS)
   17 DO20I=1,NB2
      T=B(I,I)
      B(I,I)=1.0
      DO21J=1,NB2
   21 B(I,J)=B(I,J)/T
      DO20L=1,NB2
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      IF(L-I)22,20,22
   22 T=B(L,I)
      B(L,I)=0.
      DO23J=1,NB2
   23 B(L,J)=B(L,J)-T*B(I,J)
   20 CONTINUE
      IF(IS-1)24,24,25
   25 DO26I=1,NB2
      DO26J=1,NB2
   26 TEMP(I,J)=A(J,I,IS1)
      DO27I=1,NB2
      DO27J=1,NB2
      A(I,J,IS1)=0.
      DO27L=1,NB2
   27 A(I,J,IS1)=A(I,J,IS1)&B(I,L)*TEMP(J,L)
   24 DO4L=1,NL
      DO19I=1,NB2
      TEMPV(I,L)=0.
      DO19J=1,NB2
   19 TEMPV(I,L)=TEMPV(I,L)&B(I,J)*Y(J,IS,L)
      DO4I=1,NB2
    4 Y(I,IS,L)=TEMPV(I,L)
      IF(IS-1)1,1,5
    5 DO28L=1,NL
      DO28I=1,NB2
      DO28J=1,NB2
   28 Y(I,IS1,L)=Y(I,IS1,L)-TEMP(I,J)*Y(J,IS,L)
      IS=IS-1
      GO TO 2
C     CARRY OUT BACK SUBSTITUTION
    1 DO40L=1,NL
      DO40I=1,NB2
   40 C(I,L)=Y(I,1,L)
      IS=1
   48 GO TO (45,46),IL
C     PUNCH FRAME DEFORMATIONS FOR A FLOOR.
   45 DO60L=1,NL
   60 PUNCH34,(C(I,L),I=1,NB2)
   34 FORMAT(5E14.8)
      GO TO 47
   46 PUNCH34,(C(NB2,L),L=1,NL)
   47 IF(IS-NS)41,42,42
   41 IS=IS&1
      IS1=IS-1
      DO43L=1,NL
      DO43I=1,NB2
      TEMPV(I,L)=0.
      DO43J=1,NB2
   43 TEMPV(I,L)=TEMPV(I,L)&A(I,J,IS1)*C(J,L)
      DO44L=1,NL
      DO44I=1,NB2
   44 C(I,L)=Y(I,IS,L)-TEMPV(I,L)
      GO TO 48
      END
C
      SUBROUTINE STIFFS
C     THIS SUBROUTINE CALCULATES STIFFNESS COEFFICIENTS.
      COMMON R,P,G,F,H,Q,AT,SE,SR,E
C     P=MOMENT OF INERTIA, AT=TOTAL AREA, AS=SHEAR AREA,
C     H=CENTRE LINE LENGTH, Q=CLEAR LENGTH
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      READ30,P,AT,AS,H,Q
   30 FORMAT(5E10.0)
      BETA=(6.0*P*SE)/(Q*Q*AS*SR)
      P=2.0*P*E/(Q*(1.0&2.0*BETA))
      R=P*(2.0&BETA)
      P=P*(1.0-BETA)
      G=(R&P)/(Q*Q)
      F=0.5*(H&Q)*(H-Q)
      R=R&G*F
      P=P&G*F
      F=G*H
      G=2.0*G
      AT=E*AT/Q
      RETURN
      END
```

```
C     PROGRAM ''MTDEMA''    LIST 4                              PAGE 1
C     LANGUAGE - FORTRAN PDQ FOR IBM 1620.
C     IAD=1 NO A.D., IAD=2 WITH A.D.
      DIMENSION SD(120,10)
      BEGIN TRACE
   26 READ29,NS,NB,NL,IAD,IHB,SR,SE,E
   29 FORMAT(5I3,F4.2,F4.1,E10.4)
      NB2=IAD*NB&IAD&1
      NB1=NB&1
      DO2IS=1,NS
      K1=(IS-1)*NB2&1
      K2=K1&NB2-1
C     READ DEFORMATIONS AS PUNCHED BY PROGRAM 'T.D.E.'
      DO2IY=1,NL
      DO2I=K1,K2,5
    2 READ34,SD(I,IY),SD(I&1,IY),SD(I&2,IY),SD(I&3,IY),SD(I&4,IY)
   34 FORMAT(5E14.8)
      IS=NS
C     READ BEAM DATA
   25 IU=(IS-1)*NB2
      IF(NB)27,27,28
   28 DO3I2=1,NB
      EXECUTE PROCEDURE 1
      I=IU&I2
      J=I&1
      GO TO (4,5),IAD
    4 DO6IY=1,NL
      R1=R*SD(I,IY)&P*SD(J,IY)
      R2=P*SD(I,IY)&R*SD(J,IY)
      R3=(R1&R2)/H
      R5=R1-0.5*R3*(H-Q)
      R6=R2-0.5*R3*(H-Q)
    6 PUNCH31,R1,R3,R5,R2,R6
      GO TO 3
   31 FORMAT(8E10.4)
    5 K=I&NB1
      L=K&1
      DO7IY=1,NL
      R1=R*SD(I,IY)&P*SD(J,IY)&B*(SD(K,IY)-SD(L,IY))
      R2=P*SD(I,IY)&R*SD(J,IY)&B*(SD(K,IY)-SD(L,IY))
      R3=(R1&R2)/H
      R5=R1-0.5*R3*(H-Q)
      R6=R2-0.5*R3*(H-Q)
    7 PUNCH31,R1,R3,R5,R2,R6
    3 CONTINUE
   27 IF(IHB-1)8,9,9
C     READ HALF-BEAM DATA
    9 EXECUTE PROCEDURE 1
      I=IU&NB1
      GO TO (10,11),IAD
   10 DO12IY=1,NL
      R1=B*H*SD(I,IY)
      R3=(2.0*R1)/H
      R5=R1-0.5*R3*(H-Q)
   12 PUNCH31,R1,R3,R5
      GO TO 8
   11 K=IU&2*NB1
      DO14IY=1,NL
      R1=B*H*SD(I,IY)&G*H*SD(K,IY)
      R3=(2.0*R1)/H
      R5=R1-0.5*R3*(H-Q)
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C     PROGRAM ''MTDEMA''    LIST 4                              PAGE 2
   14 PUNCH31,R1,R3,R5
C     READ COLUMN DATA
    8 IU2=IU-NB2
      DO20I2=1,NB1
      EXECUTE PROCEDURE 1
      I=IU2&I2
      J=IU
      K=IU&I2
      L=IU&NB2
      M=I&NB1
      N=K&NB1
      DO20IY=1,NL
      R1=P*SD(K,IY)-B*SD(L,IY)
      R2=R*SD(K,IY)-B*SD(L,IY)
      R3=(R1&R2)/H
      R5=R1-0.5*R3*(H-Q)
      R6=R2-0.5*R3*(H-Q)
      GO TO (18,19),IAD
   19 R4=-AT*SD(N,IY)
      IF(IU)24,24,21
   18 IF(IU)22,22,21
   21 R1=R1&R*SD(I,IY)&B*SD(J,IY)
      R2=R2&P*SD(I,IY)&B*SD(J,IY)
      R3=(R1&R2)/H
      R5=R1-0.5*R3*(H-Q)
      R6=R2-0.5*R3*(H-Q)
      GO TO (22,23),IAD
   23 R4=R4&AT*SD(M,IY)
   24 PUNCH31,R1,R3,R5,R2,R6,R4
      GO TO 20
   22 PUNCH31,R1,R3,R5,R2,R6
   20 CONTINUE
      IS=IS-1
      IF(IS)26,26,25
      BEGIN PROCEDURE 1
C     P=MOMENT OF INERTIA, AT=TOTAL AREA, AS=SHEAR AREA,
C     H=CENTRE LINE LENGTH, Q=CLEAR LENGTH
      READ30,P,AT,AS,H,Q
   30 FORMAT(5E10.0)
      BETA=(6.0*P*SE)/(Q*Q*AS*SR)
      P=2.0*P*E/(Q*(1.0&2.0*BETA))
      R=P*(2.0&BETA)
      P=P*(1.0-BETA)
      G=(R&P)/(Q*Q)
      F=0.5*(H-Q)*(H&Q)
      R=R&G*F
      P=P&G*F
      B=G*H
      G=2.0*G
      AT=E*AT/Q
      END PROCEDURE 1
      END TRACE
      END
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C     LANGUAGE - FORTRAN II FOR IBM 360/44
C
C     THIS PROGRAM CALCULATES THE LATERAL STIFFNESS AND
C     FLEXIBILITY MATRICES FOR A REGULAR MULTI-STOREY FRAME
C     UP TO 15 STOREYS BY SIX BAYS WITH A.D.,13 BAYS NO A.D.,
C     SHEAR DEFORMATION AND JOINT SIZE ALSO CONSIDERED.
C     IAD=2 CONSIDERS A.D., IAD=1 IGNORES A.D.
C     READ MEMBER DATA BOTTOM STOREY TO TOP
      DIMENSION A(15,15),B(15,15),C(15,15),D(15,15),E(15,15),F(15,15),
     1G(15,15),H(15,15),Z(15,15)
      COMMON SE,SR,EE,R,P,GG,FF,HH,Q,AT
C     NS=NO OF STOREYS, NB=NO OF BAYS, IHB=1 FOR HALF BAY,
C     SR=RATIO OF G/E (.45 FOR R.C.),SE=EFFECTIVE SHEAR
C     AREA FACTOR (1.2 FOR RECT.), EE=YOUNGS MODULUS.
C     KS1=1 TO PUNCH(K)LAT,2 TO SKIP.
C     KS3=1 TO PUNCH(F)LAT,2 TO SKIP
   21 READ30,NS,NB,IAD,IHB,SR,SE,EE,KS1,KS3
   30 FORMAT(4I3,F4.2,F4.1,E10.4,2I3)
      PRINT130,NS,NB,IAD,IHB,SR,SE,EE,KS1,KS3
  130 FORMAT(1X,4I4,F4.2,F4.1,E13.4,2I4)
      NU=IAD*(NB&1)
      NB1=NB&1
      DO43I=1,NU
      DO44J=1,NU
   44 E(I,J)=0.
      DO43J=1,NS
      F(I,J)=0.
   43 H(J,I)=0.
      DO51I=1,NS
      DO51J=1,NS
   51 Z(I,J)=0.
      NS1=NS&1
      DO22IS=1,NS1
      IF(IS-NS1)1,14,14
C     READ COLUMN DATA
    1 DO7I=1,NB1
      CALL STIFFS
      E(I,I)=E(I,I)&R
      GO TO (8,9),IAD
    9 K=I&NB1
      E(K,K)=E(K,K)&AT
    8 F(I,IS)=F(I,IS)-FF
      H(IS,I)=F(I,IS)
      Z(IS,IS)=Z(IS,IS)&GG
      IF(IS-1)7,7,10
   10 A(I,I)=A(I,I)&R
      B(I,I)=B(I,I)&P
      D(I,I)=B(I,I)
      GO TO (12,11),IAD
   11 A(K,K)=A(K,K)&AT
      B(K,K)=B(K,K)-AT
      D(K,K)=B(K,K)
   12 IS1=IS-1
      C(I,IS1)=C(I,IS1)&FF
      G(IS1,I)=C(I,IS1)
      C(I,IS)=C(I,IS)-FF
      G(IS,I)=C(I,IS)
      F(I,IS1)=F(I,IS1)&FF
      H(IS1,I)=F(I,IS1)
      Z(IS1,IS1)=Z(IS1,IS1)&GG
```



```
      Z(IS1,IS)=Z(IS1,IS)-GG
      Z(IS,IS1)=Z(IS1,IS)
    7 CONTINUE
C     READ BEAM DATA
      IF(NB)902,902,903
  903 DO3I=1,NB
      CALL STIFFS
      I1=I&1
      GO TO (45,46),IAD
   46 K=NB1&I
      K1=K&1
      E(I,K)=E(I,K)&FF
      E(K,I)=E(I,K)
      E(I,K1)=E(I,K1)-FF
      E(K1,I)=E(I,K1)
      E(I1,K)=E(I1,K)&FF
      E(K,I1)=E(I1,K)
      E(I1,K1)=E(I1,K1)-FF
      E(K1,I1)=E(I1,K1)
      E(K,K)=E(K,K)&GG
      E(K,K1)=E(K,K1)-GG
      E(K1,K)=E(K,K1)
      E(K1,K1)=E(K1,K1)&GG
   45 E(I,I)=E(I,I)&R
      E(I,I1)=E(I,I1)&P
      E(I1,I)=E(I,I1)
    3 E(I1,I1)=E(I1,I1)&R
  902 IF(IHB-1)4,5,5
C     READ HALF-BEAM DATA, FULL LENGTH, ACTUAL I
    5 CALL STIFFS
      E(NB1,NB1)=E(NB1,NB1)&FF*HH
      GO TO (5,6),IAD
    6 E(NB1,NU)=E(NB1,NU)&GG*HH
      E(NU,NB1)=E(NB1,NU)
      E(NU,NU)=E(NU,NU)&2.0*GG
    4 IF(IS-1)17,17,14
   14 CALL MATINV(A,NU)
      CALL MTPCOP(A,C,NU,NS)
      IF(IS-NS1)15,16,16
   15 CALL MTPCOP(A,B,NU,NU)
      CALL MTPDFC(D,B,E,NU,NU,NU)
      CALL MTPDFC(D,C,F,NU,NU,NS)
      CALL MTPDFC(G,B,H,NS,NU,NU)
   16 CALL MTPDFC(G,C,Z,NS,NU,NS)
      IF(IS-NS1)17,18,18
   18 DO52I=1,NS
   52 PRINT33,(Z(I,J),J=1,NS)
   33 FORMAT(1X,10E13.4)
      GO TO (26,27),KS1
   26 DO900 I=1,NS
  900 PUNCH39,(Z(I,J),J=1,NS)
   39 FORMAT(5E14.7)
   27 CALL MATINV(Z,NS)
      DO53I=1,NS
   53 PRINT33,(Z(I,J),J=1,NS)
      GO TO (28,22),KS3
   28 DO 901 I=1,NS
  901 PUNCH39,(Z(I,J),J=1,NS)
      STOP 111
   17 DO25I=1,NU
```

```
      DO50J=1,NU
      A(I,J)=E(I,J)
      E(I,J)=0.
      B(I,J)=0.
   50 D(I,J)=0.
      DO25J=1,NS
      C(I,J)=F(I,J)
      F(I,J)=0.
      G(J,I)=H(J,I)
   25 H(J,I)=0.
   22 CONTINUE
      STOP 1111
      END
      SUBROUTINE MATINV(A,N)
C     (A)=(A)-1 ORDER N
      DIMENSION A(15,15)
      DO1I=1,N
      T=A(I,I)
      IF(T.NE.0.) GO TO 20
      PRINT33,I
   33 FORMAT(16H SINGULAR MATRIX,I3)
      STOP 999
   20 A(I,I)=1.0
      DO2J=1,N
    2 A(I,J)=A(I,J)/T
      DO1L=1,N
      IF(L-I)3,1,3
    3 T=A(L,I)
      A(L,I)=0.
      DO4J=1,N
    4 A(L,J)=A(L,J)-T*A(I,J)
    1 CONTINUE
      RETURN
      END
      SUBROUTINE STIFFS
C     SUBROUTINE TO CALCULATE STIFF. COEFF. FOR (K)LAT
      COMMON SE,SR,E,R,P,G,F,H,Q,AT,AS,BETA
C     P=MOMENT OF INERTIA, AT=TOTAL AREA, AS=SHEAR AREA,
C     H=CENTRE LINE LENGTH, Q=CLEAR LENGTH
      READ30,P,AT,AS,H,Q
   30 FORMAT(5E10.4)
      PRINT31,P,AT,AS,H,Q
   31 FORMAT(1X,10E13.4)
      BETA=(6.0*P*SE)/(Q*Q*AS*SR)
      P=2.0*P*E/(Q*(1.0&2.0*BETA))
      R=P*(2.0&BETA)
      P=P*(1.0-BETA)
      G=(R&P)/(Q*Q)
      F=0.5*(H&Q)*(H-Q)
      R=R&G*F
      P=P&G*F
      F=G*H
      G=2.0*G
      AT=E*AT/Q
      RETURN
      END
      SUBROUTINE MTPDFC(A,B,C,L,M,N)
C     (C)=(C)-(A)*(B).  A ORDER LXM, B ORDER MXN
      DIMENSION A(15,15),B(15,15),C(15,15)
      DO1I=1,L
```



```
      DO1K=1,N
      WORK=0.
      DO2J=1,M
    2 WORK=WORK&A(I,J)*B(J,K)
    1 C(I,K)=C(I,K)-WORK
      RETURN
      END
      SUBROUTINE MTPCOP(A,B,M,N)
C     (B)=(A)*(B) A ORDER MXM, B ORDER MXN
      DIMENSION A(15,15),B(15,15),T(15,15)
      DO1I=1,M
      DO1K=1,N
      WORK=0.
      DO2J=1,M
    2 WORK=WORK&A(I,J)*B(J,K)
    1 T(I,K)=WORK
      DO3I=1,M
      DO3J=1,N
    3 B(I,J)=T(I,J)
      RETURN
      END
```

```
C      PROGRAM ''INSTAB''    LIST 6                          PAGE 1
C      LANGUAGE - FORTRAN IV FOR IBM 360/44
C      INSTABILITY OF REGULAR MULTI-STOREY FRAMES.
       DIMENSION A(11,11,14),B(11,11),C(11,11),Y(11,15,1),Z(11,15,1),
      1TEMP(11,11),TEMPV(11,1),AL2(5),AL3(5)
       COMMON PP(30),QQ(30),AT(21),AL(18),R,P,G,F,H,E,I,IS,NB1,IBC,KK,MMM
C      NS=NO OF STOREYS, NB=NO OF BAYS, IHB=1 FOR HALF-BAY,NL=NO OF LOADS
C      E= YOUNG'S MODULUS
       READ29,NS,NB,IHB,E
    29 FORMAT(3I3,E10.4)
       PRINT36,NS,NB,IHB,E
    36 FORMAT(1X,3I4,E13.4)
       PRINT37
    37 FORMAT(2X)
       NL=1
       NB2=2*NB&3
       NS1=NS-1
       NB1=NB&1
       J=NB1*NS&1
       K=J&NB1
       DO83I=J,K
    83 AT(I)=0.
C      READ AND STORE MEMBER DATA, TOP STOREY TO BOTTOM, LEFT TO RIGHT.
C      PP=MOMENT OF INERTIA    QQ=LENGTH    INCH UNITS.
       K=NS*(NB*2&1)
       DO51M=1,K
       READ33,PP(M),QQ(M)
    51 PRINT34,PP(M),QQ(M)
    34 FORMAT(1X,10E13.4)
       PRINT37
C      READ APPLIED LOADS
       DO77K=1,NS
       READ33,(Z(J,K,1),J=1,NB2)
    77 PRINT34,(Z(J,K,1),J=1,NB2)
    33 FORMAT(8E10.4)
C      FFF=FACTOR ON ALL LOADS, EXCEPT TOP STOREY LATERAL LOAD
C      WHICH IS KEPT CONSTANT.
    76 READ33,FFF
       PRINT34,FFF
C      INITIALISE AXIAL LOADS.
       DO61I=1,NB1
    61 AL2(I)=0.
       J=NS
    62 DO63I=1,NB1
       L=NB1&I
       K=NB1*J-NB1&I
       AL(K)=AL2(I)-Z(L,J,1)
    63 AL2(I)=AL(K)
       J=J-1
       IF(J.GT.0) GO TO 62
    68 K=NS*NB1
       PRINT38,(AL(I),I=1,K)
    38 FORMAT(/(1X,10E13.4))
       DO64I=1,NB2
       DO64J=1,NS
    64 Y(I,J,1)=Z(I,J,1)*FFF
       Y(NB2,NS,1)=Z(NB2,NS,1)
       MMM=0
       IF=1
       IS=NS
C      SET UP STOREY ELEMENT STIFFNESS MATRICES.
       DO3I=1,NB2
       DO3J=1,NB2
       B(I,J)=0.
```

```
C      PROGRAM ''INSTAB''    LIST 6                          PAGE 2
       C(I,J)=0.
       DO3K=1,NS1
     3 A(I,J,K)=0.
     2 DO50I=1,NB2
       DO50J=1,NB2
       B(I,J)=C(I,J)
    50 C(I,J)=0.
       IS1=IS-1
C      TAKE BEAM DATA LEFT TO RIGHT AND CALCULATE STIFFNESS COEFFICIENTS.
       IBC=1
       DO80I=1,NB
       MMM=MMM&1
       CALL STIFFS
       I1=I&1
       B(I,I)=B(I,I)&R
       B(I,I1)=B(I,I1)&P
       B(I1,I)=B(I,I1)
       B(I1,I1)=B(I1,I1)&R
    81 K=NB1&I
       K1=K&1
       B(I,K)=B(I,K)&F
       B(K,I)=B(I,K)
       B(I,K1)=B(I,K1)-F
       B(K1,I)=B(I,K1)
       B(K,K)=B(K,K)&G
       B(K,I1)=B(K,I1)&F
       B(I1,K)=B(K,I1)
       B(K,K1)=B(K,K1)-G
       B(K1,K)=B(K,K1)
       B(I1,K1)=B(I1,K1)-F
       B(K1,I1)=B(I1,K1)
       B(K1,K1)=B(K1,K1)&G
    80 CONTINUE
       IF(IHB-1)14,15,15
C      TAKE HALF-BEAM DATA AND CALC. STIFFNESS COEFF.
    15 MMM=MMM&1
       CALL STIFFS
       B(NB1,NB1)=B(NB1,NB1)&F*H
    52 J=NB1*2
       B(NB1,J)=B(NB1,J)&G*H
       B(J,NB1)=B(NB1,J)
       B(J,J)=B(J,J)&2.0*G
C      TAKE COLUMN DATA LEFT TO RIGHT AND CALCULATE STIFFNESS COEFF.
    14 J=NB2
       IBC=2
       DO12I=1,NB1
       MMM=MMM&1
       CALL STIFFS
       B(I,I)=B(I,I)&R
       B(I,J)=B(I,J)-F
       B(J,I)=B(I,J)
       B(J,J)=B(J,J)&G
       K=NB1&I
       B(K,K)=B(K,K)&AT(KK)
     6 IF(IS-1)12,12,11
    11 C(I,I)=C(I,I)&R
       C(I,J)=C(I,J)&F
       C(J,I)=C(I,J)
       C(J,J)=C(J,J)&G
     9 C(K,K)=C(K,K)&AT(KK)
     8 A(I,I,IS1)=A(I,I,IS1)&P
       A(I,J,IS1)=A(I,J,IS1)&F
       A(J,I,IS1)=A(J,I,IS1)-F
```

```
      A(J,J,IS1)=A(J,J,IS1)-G
   10 A(K,K,IS1)=A(K,K,IS1)-AT(KK)
   12 CONTINUE
C     REDUCE (K) TO LOWER DIAGONAL FORM BY ELIMINATION
      IF(IS-NS)16,17,17
   16 DO18L=1,NB2
      DO18I=1,NB2
      DO18J=1,NB2
   18 B(I,L)=B(I,L)-TEMP(I,J)*A(J,L,IS)
   17 DO20I=1,NB2
      T=B(I,I)
      B(I,I)=1.0
      DO21J=1,NB2
   21 B(I,J)=B(I,J)/T
      DO20L=1,NB2
      IF(L-I)22,20,22
   22 T=B(L,I)
      B(L,I)=0.
      DO23J=1,NB2
   23 B(L,J)=B(L,J)-T*B(I,J)
   20 CONTINUE
      IF(IS-1)24,24,25
   25 DO26I=1,NB2
      DO26J=1,NB2
   26 TEMP(I,J)=A(J,I,IS1)
      DO27I=1,NB2
      DO27J=1,NB2
      A(I,J,IS1)=0.
      DO27L=1,NB2
   27 A(I,J,IS1)=A(I,J,IS1)&B(I,L)*TEMP(J,L)
   24 DO4L=1,NL
      DO19I=1,NB2
      TEMPV(I,L)=0.
      DO19J=1,NB2
   19 TEMPV(I,L)=TEMPV(I,L)&B(I,J)*Y(J,IS,L)
      DO4I=1,NB2
    4 Y(I,IS,L)=TEMPV(I,L)
      IF(IS-1)1,1,5
    5 DO28L=1,NL
      DO28I=1,NB2
      DO28J=1,NB2
   28 Y(I,IS1,L)=Y(I,IS1,L)-TEMP(I,J)*Y(J,IS,L)
      IS=IS-1
      GO TO 2
C     CARRY OUT BACK SUBSTITUTION
    1 DO40L=1,NL
      DO40I=1,NB2
   40 C(I,L)=Y(I,1,L)
      DO65I=1,NB1
   65 AL2(I)=0.
      DO44IS=1,NS
      PRINT34,(C(I,1),I=1,NB2)
   66 DO72I=1,NB1
      J=(IS-1)*NB1&I
      L=IS*NB1&I
      K=NB1&I
      AL3(I)=-AT(J)*C(K,1)&AL2(I)
      AL2(I)=AT(L)*C(K,1)
      IF(AL(J))100,71,100
  100 IF(AL3(I)/AL(J)-1.001)46,46,71
   46 IF(AL3(I)/AL(J)-.999)71,72,72
   71 IE=2
   72 AL(J)=AL3(I)
```

LIST 6


```
   47 IF(IS-NS)41,69,69
   41 IS1=IS&1
      DO43L=1,NL
      DO43I=1,NB2
      TEMPV(I,L)=0.
      DO43J=1,NB2
   43 TEMPV(I,L)=TEMPV(I,L)&A(I,J,IS)*C(J,L)
      DO44L=1,NL
      DO44I=1,NB2
   44 C(I,L)=Y(I,IS1,L)-TEMPV(I,L)
   69 GO TO (73,68),IE
   73 P=Z(NB2,NS,1)/C(NB2,1)
  169 PRINT38,P
   42 PRINT35
   35 FORMAT(' PROCESSING COMPLETE'//)
      GO TO 76
      END
      SUBROUTINE STIFFS
      DIMENSION AE(7)
      COMMON PP(30),QQ(30),AT(21),AL(18),R,P,G,F,Q,E,I,IS,NB1,IBC,K,M
      P=PP(M)
      Q=QQ(M)
      A=1.E10
      PM=P
      GO TO (1,2),IBC
    1 T1=1.0
      T2=1.0
      T3=1.0
      T4=1.0
      GO TO 3
    2 K=NB1*IS-NB1&I
      AT(K)=E*A/Q
      R=-AL(K)*Q*Q/(9.8696044*E*PM)
      IF(ABS(R)-4.)9,9,10
   10 PRINT31,M,R,E,Q,AL(K)
   31 FORMAT(' AXIAL LOAD EXCEEDS ALLOWABLE RANGE.',I4,4E13.4)
      STOP22222
    9 IF(R-.001)5,5,6
    5 IF(-R-.001)1,1,6
    6 AE(1)=1.5797363
      AE(2)=0.15858587
      AE(3)=0.02748899
      AE(4)=0.00547540
      AE(5)=0.00115281
      AE(6)=0.00024908
      AE(7)=0.00005452
      SUM=0.
      DO4L=1,7
    4 SUM=SUM&AE(L)*R**L/2.**(3*L)
      WP=(64.-60.*R&5.*R*R)/(64.-20.*R&R*R)-SUM
      SUM=(9.8696044*R*.25)/(1.-WP)
      S=SUM&WP
      C=(SUM-WP)/S
    8 T2=S*(1.0&C)*.16666667
      T1=T2&AL(K)*Q*Q*.083333333/(E*P)
      T3=.25*S
      T4=.50*S*C
    3 R=4.0*P*E*T3/Q
      F=6.0*P*E*T2/(Q*Q)
      G=12.0*P*E*T1/(Q*Q*Q)
      P=2.0*P*E*T4/Q
      RETURN
      END
```

CHAPTER THREE

DYNAMIC ELASTIC RESPONSE OF MULTI-MASS SYSTEMS

3.1 DETERMINATION OF ELASTIC RESPONSE BY NUMERICAL INTEGRATION

The elastic response of multi-mass structures to earthquake ground motion may be found by the direct integration of the equations of motion using a digital record to represent the earthquake.

If the structure is idealised so that its mass is lumped at discrete points then the mass properties are assumed to be separated from the elastic characteristics and the dynamic equilibrium equations may be written as a finite number of ordinary differential equations which may be expressed most conveniently in matrix form:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]_{LAT}\{X\} = -\ddot{X}_g [M]$$

where $\{X\}$ = Lateral displacements of the masses of the structure,

$\{\dot{X}\}$ = Lateral velocities of the masses of the structure,

$\{\ddot{X}\}$ = Lateral accelerations of the masses of the structure,

$\{X\}$, $\{\dot{X}\}$ and $\{\ddot{X}\}$ are measured relative to the base of the structure,

$\ddot{X}_g$ is the acceleration of the ground

$[M]$ is a diagonal matrix of masses lumped at discrete points,

$[C]$ is the damping matrix,

$[K]_{LAT}$ is the lateral stiffness matrix of the structure.

There are two approaches to the solution of this differential equation. The first involves the direct numerical integration of the equilibrium equations in their original form, usually by a step-by-step process giving the response in terms of the displacement of each of the storey masses. The second method is the mode superposition technique which involves the solution of the eigenvalue problem to give the normal mode properties. The equations of motion are transformed so that the system is uncoupled and the response of each mode is determined independently. The total response is obtained by combining the modal responses at each step using the characteristic properties of each mode.

The direct integration of the equations of motion in their original form is more general then the normal mode superposition method, because there is no restriction on the magnitude or form of the damping present, and it may be applied to non-linear systems by modifying the assumed linear properties at each successive step of integration, as will be shown later.

However if the damping present is small and in a certain restricted form, the integration of the modal equations of motion possesses certain advantages. If the modal responses are combined after each step of integration there is no uncertainty about the magnitude and time of occurrence of the maximum response, as with the determination of the response of each mode from a response spectrum when some approximation such as the root-mean-square technique must be used to calculate the total response. The main advantage is that usually the major part of the response of the structure occurs in the first few modes of vibration and so it is necessary only to integrate a few independent differential equations to obtain the response. Before this can be done it is necessary to carry out a modal analysis.

The development of a step-by-step direct integration method is now outlined.

If it is assumed that the acceleration associated with each degree of freedom varies linearly within a time interval Δt, expressions may be derived(9) for the velocity and displacement at the end of the time interval.

$$\{\dot{X}\}_t = \{\dot{X}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{X}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{X}\}_t$$

$$\{X\}_t = \{X\}_{t-\Delta t} + \Delta t\{\dot{X}\}_{t-\Delta t} + \frac{\Delta t^2}{3}\{\ddot{X}\}_{t-\Delta t} + \frac{\Delta t^2}{6}\{\ddot{X}\}_t$$

where subscript t denotes the response at time t and $(t - \Delta t)$ that at time $(t - \Delta t)$.

Although these expressions are approximations because of the assumption of linearity of acceleration, they are sufficiently accurate if Δt is small.

If these expressions are substituted into the original equation the value of $\{\ddot{X}_t\}$ is found to be

$$\{\ddot{X}\}_t = [F]\left\{-\ddot{X}_g[M] - [C]\{a\} - [K]\{b\}\right\}$$

where $[F] = \left[[M] + \frac{\Delta t}{2}[C] + \frac{\Delta t^2}{6}[K]\right]^{-1}$

$$\{a\} = \{\dot{X}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{X}\}_{t-\Delta t}$$

and $\{b\} = \{X\}_{t-\Delta t} + \Delta t\{\dot{X}\}_{t-\Delta t} + \frac{\Delta t^2}{3}\{\ddot{X}\}_{t-\Delta t}$

Hence the numerical integration may be carried out as follows:

$\{X\}_0$ and $\{\dot{X}\}_0$ are known as initial conditions,

$\{\ddot{X}\}_0$ is determined from the original equation,

i.e.

$$\{\ddot{X}\}_0 = [M]^{-1}\left\{-\ddot{X}_g[M] - [C]\{\dot{X}\}_0 - [K]\{X\}_0\right\}$$

The initialisation is then completed by calculating $[F]$

$$[F] = \left[[M] + \frac{\Delta t}{2}[C] + \frac{\Delta t^2}{6}[K]\right]^{-1}$$

The following calculations are then made repeatedly, giving the response at the end of each step interval:

$$\{a\} = \{\dot{X}\}_{t-\Delta t} + \frac{\Delta t}{2}\{\ddot{X}\}_{t-\Delta t}$$

$$\{b\} = \{X\}_{t-\Delta t} + \Delta t\{\dot{X}\}_{t-\Delta t} + \frac{\Delta t^2}{3}\{\ddot{X}\}_{t-\Delta t}$$

$$\{\ddot{X}\}_t = [F]\Big\{-\ddot{X}_g[M] - [C]\{a\} - [K]\{b\}\Big\}$$

$$\{\dot{X}\}_t = \{a\} + \frac{\Delta t}{2}\{\ddot{X}\}_t$$

$$\{X\}_t = \{b\} + \frac{\Delta t^2}{6}\{\ddot{X}\}_t$$

A computer program "ELRES" has been written based on the above method and is shown in List 7, with the block diagram in Figure 3.7. The program requires as input the matrices $[K]_{LAT}$ and $[C]$. The former is calculated by the program "KLAT" detailed previously and the calculation of the latter is described in a following section. The program is dimensioned so that up to a 20 mass system may be analysed.

The method may be used to determine the response of one degree of freedom systems; all the matrices detailed above are then of order one. The integration procedure may then be used to determine response spectra and multi-mass response by mode superposition.

3.2 DETERMINATION OF RESPONSE SPECTRA

The response spectra for a given earthquake may be conveniently determined by calculating the maximum response of a series of one degree of freedom resonators with a suitable range of periods and damping. A one degree of freedom response program "SPECTRA" shown in List 8 was written to integrate the equations of motion for a series of resonators, and response spectra for the N-S component of the 1940 El Centro earthquake were determined and also for the 1966 Cholame Shandon California earthquake. The first record, shown in Figure 3.1, was supplied in corrected form by Professor G.V. Be[illegible] of Michigan University, and the latter record, shown in Figure 3.2, was supplied by Mr. R.M. Thompson of Auckland University, and was integrated in uncorrected form. The response spectrum for these two records is shown in Figures 3.3 to 3.6.

3.3 RESPONSE BY NUMERICAL INTEGRATION USING MODE SUPERPOSITION

If the response of a multi-mass system is to be found by superimposing the modal responses then the equations of motion of the normal modes must be integrated independently as a series of one degree of freedom systems. A program "SUMMOD", shown in List 9, has been written which accepts the fraction of critical damping, the periods of vibration and the 1g displacements for each mode and calculates the displacement of each resonator in the form of the spectral acceleration; i.e. displacement times the angular frequency squared. The total response of the structure is found by determining the amount of the displacement in each mode by multiplying the 1g displacements by the spectral acceleration, and summing the resulting modal displacements algebraically. This is done at the end of each step and so the magnitude and time of occurrence of maximum response can be found.

3.4 CALCULATION OF THE DAMPING MATRIX [C]

The damping has been assumed to be of viscous form. This assumption has not been strictly justified, either experimentally or theoretically, although it has been commonly made and it has been shown that most forms of damping can be converted into an equivalent viscous form[20].

For the normal modes to be independent of one another it is necessary that the damping matrix $[C]$ satisfies the following condition[21]

$$[\Phi]^T [C][\Phi] = [2\omega_r \bar{C}_r]$$

where $[\Phi]$ is a square matrix, the rth column ϕ_r being the displacement ratios of the rth mode normalised such that

$$\{\phi_r\}^T [M]\{\phi_r\} = 1.0$$

and ω_r is the angular frequency of mode r

$\bar{C}_r$ is the fraction of critical damping in mode r.

Hence the matrix $[C]$ may be calculated if a modal analysis is first carried out to determine ω_r and $\{\phi_r\}$ for every mode, and if $\bar{C}_r$ is specified for every mode.

Then

$$[C] = [\Phi^T]^{-1} [2\omega_r \bar{c}_r] [\Phi]^{-1}$$

3.5 SIMPLIFIED CALCULATION OF [C]

The previous approach requires the calculation of all the normal mode frequencies. For many buildings only the first few modes are significant and this simplified approach merely specifies the fraction of critical damping in the first two modes and also the distribution of dampers.

Two sets of dampers are assumed, one associated with the lateral stiffness and the other with the masses. The former are given coefficients proportional to the corresponding storey stiffness and the latter proportional to the storey mass, i.e. the equations of motion are assumed to be of the form:

$$[M]\{\ddot{X}\} + c_g [K]_{LAT} \{\dot{X}\} + c_t [M]\{\dot{X}\} + [K]_{LAT}\{X\} = 0$$

These equations may be transformed and shown to be uncoupled.

C_g and C_t are coefficients which may be adjusted to give any desired fraction of damping in the first two modes.

Because(21)

$$[\Phi]^T [M][\Phi] = [I] \quad \text{the unit diagonal matrix}$$

and $$[\Phi]^T [K]_{LAT}[\Phi] = [W^2]$$

where $\left[W^2 \right]$ is a diagonal matrix of the angular frequencies squared,

then

$$C_g \left[W^2 \right] + C_t = \left[2\bar{C}_r W_r \right]$$ as the modes are independent of one another.

The coefficients C_g and C_t may then be calculated if the fractions of critical damping in the first two modes are specified.

Then

$$C_g W_1^{\;2} + C_t = 2\bar{C}_1 W_1$$

$$C_g W_2^{\;2} + C_t = 2\bar{C}_2 W_2$$

$$C_g = 2 \frac{\bar{C}_2 W_2 - \bar{C}_1 W_1}{W_2^{\;2} - W_1^{\;2}}$$

$$C_t = 2\bar{C}_1 W_1 - C_g W_1^{\;2}$$

and

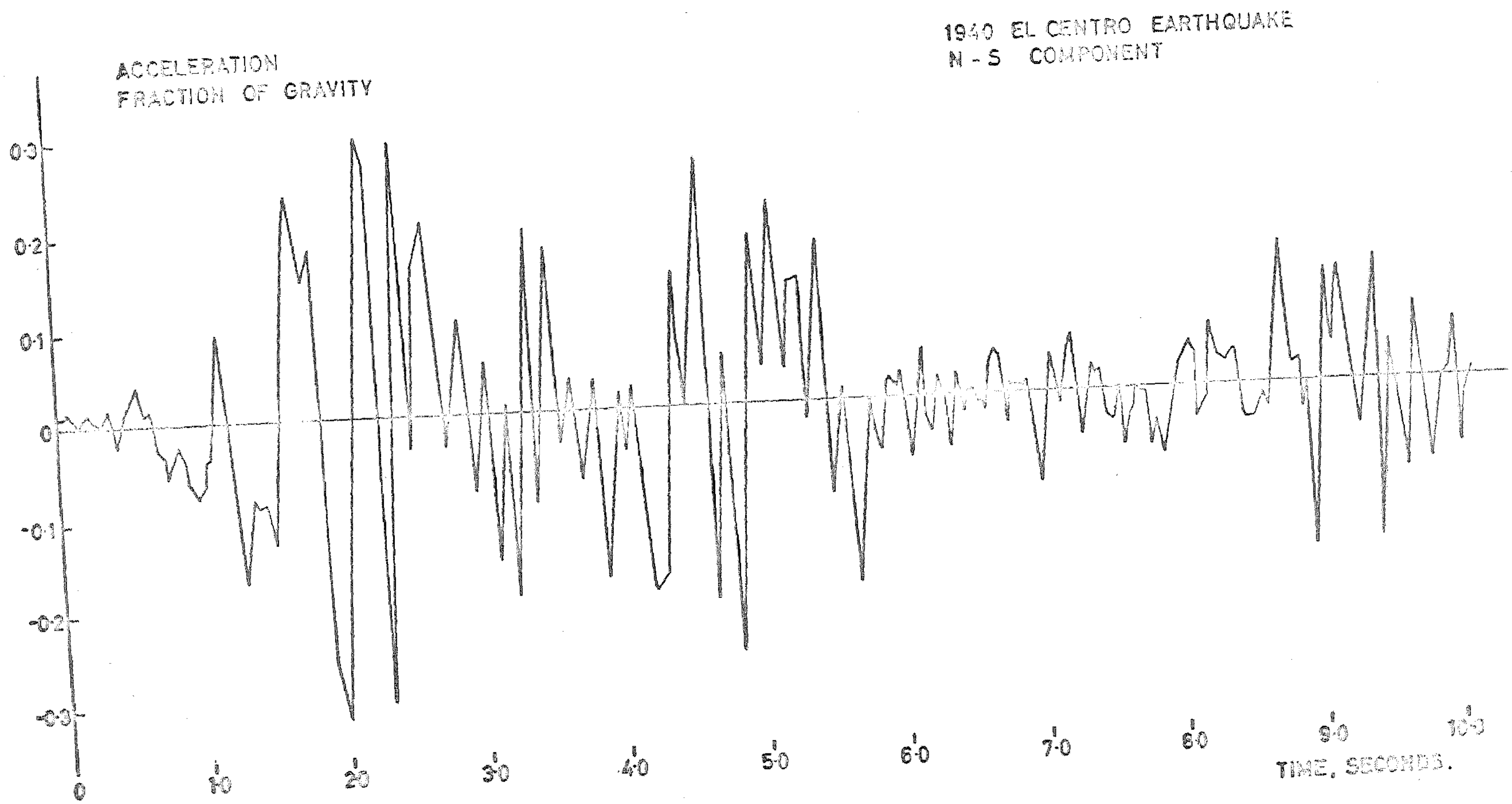
$$\left[C \right] = C_g \left[K \right]_{LAT} + C_t \left[M \right]$$

The fractions of critical damping in the third and higher modes are not specified, but may be calculated by carrying out the following operations on the simplified damping matrix.

$$\left[2W_r \bar{C}_r \right] = \left[\Phi \right]^T \left[C \right] \left[\Phi \right]$$

The fractions of critical damping in the higher modes were found to be a little higher than those specified for the first two modes for those matrices checked.

This second simplified method of calculating the damping matrix was used for all the frames discussed in this thesis.



1940 EL CENTRO EARTHQUAKE
N - S COMPONENT
ACCELERATION
FRACTION OF GRAVITY
0·3
0·2
0·1
0
-0·1
-0·2
-0·3
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 3·1

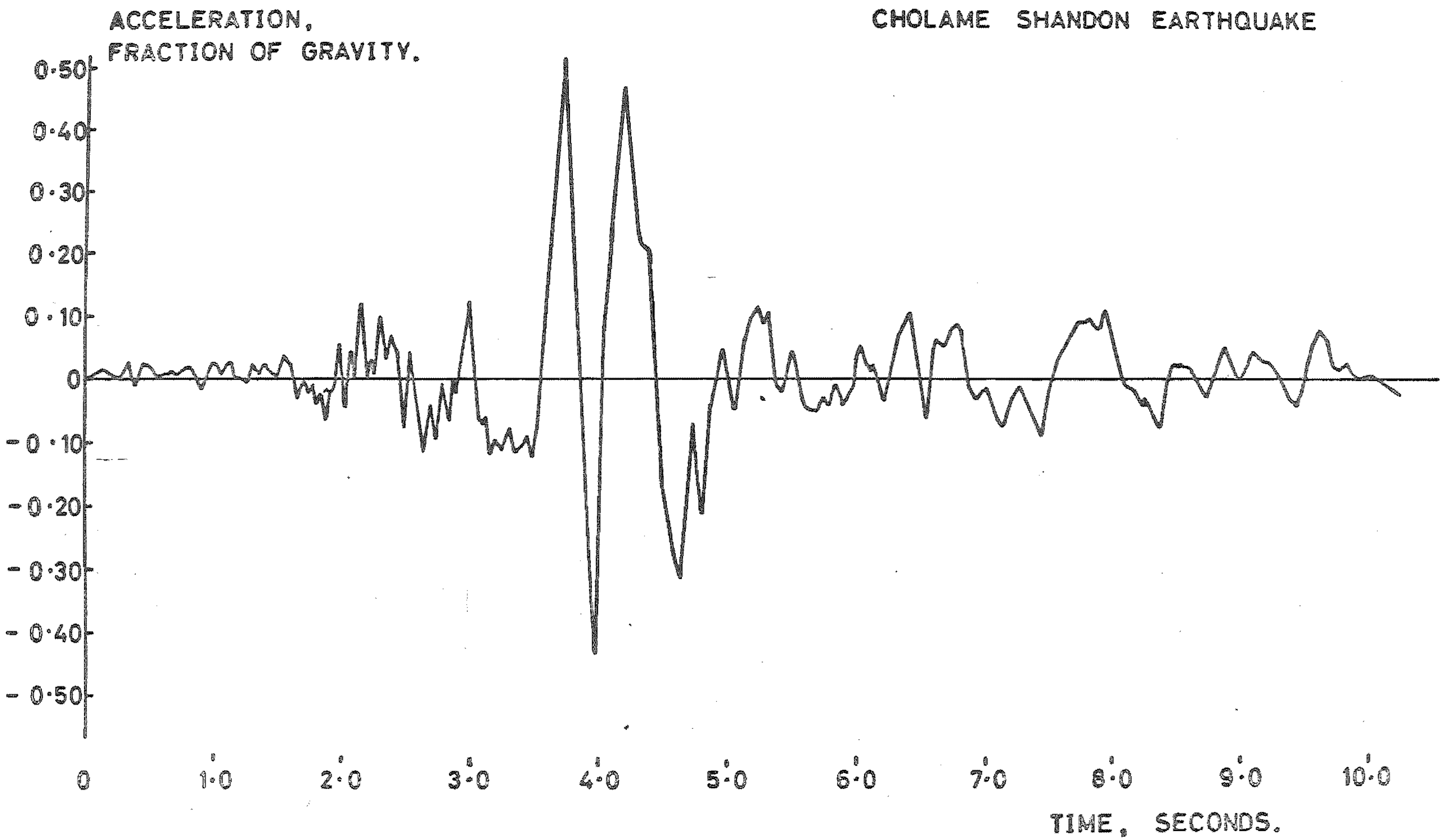

ACCELERATION,
FRACTION OF GRAVITY.
CHOLAME SHANDON EARTHQUAKE
0·50
0·40
0·30
0·20
0·10
0
-0·10
-0·20
-0·30
-0·40
-0·50
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 3·2

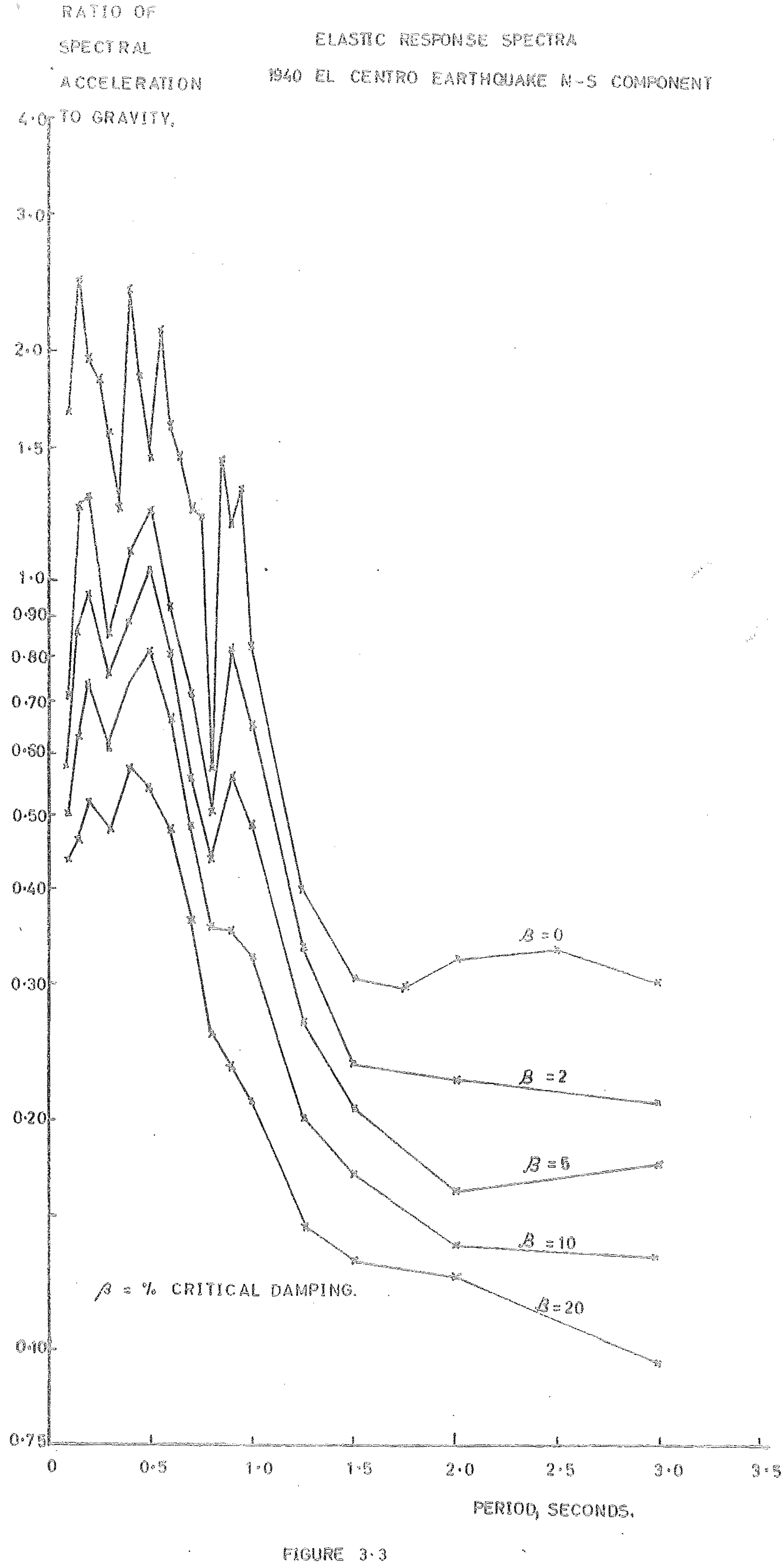

RATIO OF SPECTRAL ACCELERATION TO GRAVITY.
ELASTIC RESPONSE SPECTRA
1940 EL CENTRO EARTHQUAKE N-S COMPONENT
4·0
3·0
2·0
1·5
1·0
0·90
0·80
0·70
0·60
0·50
0·40
0·30
0·20
0·10
0·75
0
0·5
1·0
1·5
2·0
2·5
3·0
3·5
PERIOD, SECONDS.
β = 0
β = 2
β = 5
β = 10
β = 20
β = % CRITICAL DAMPING.

FIGURE 3·3

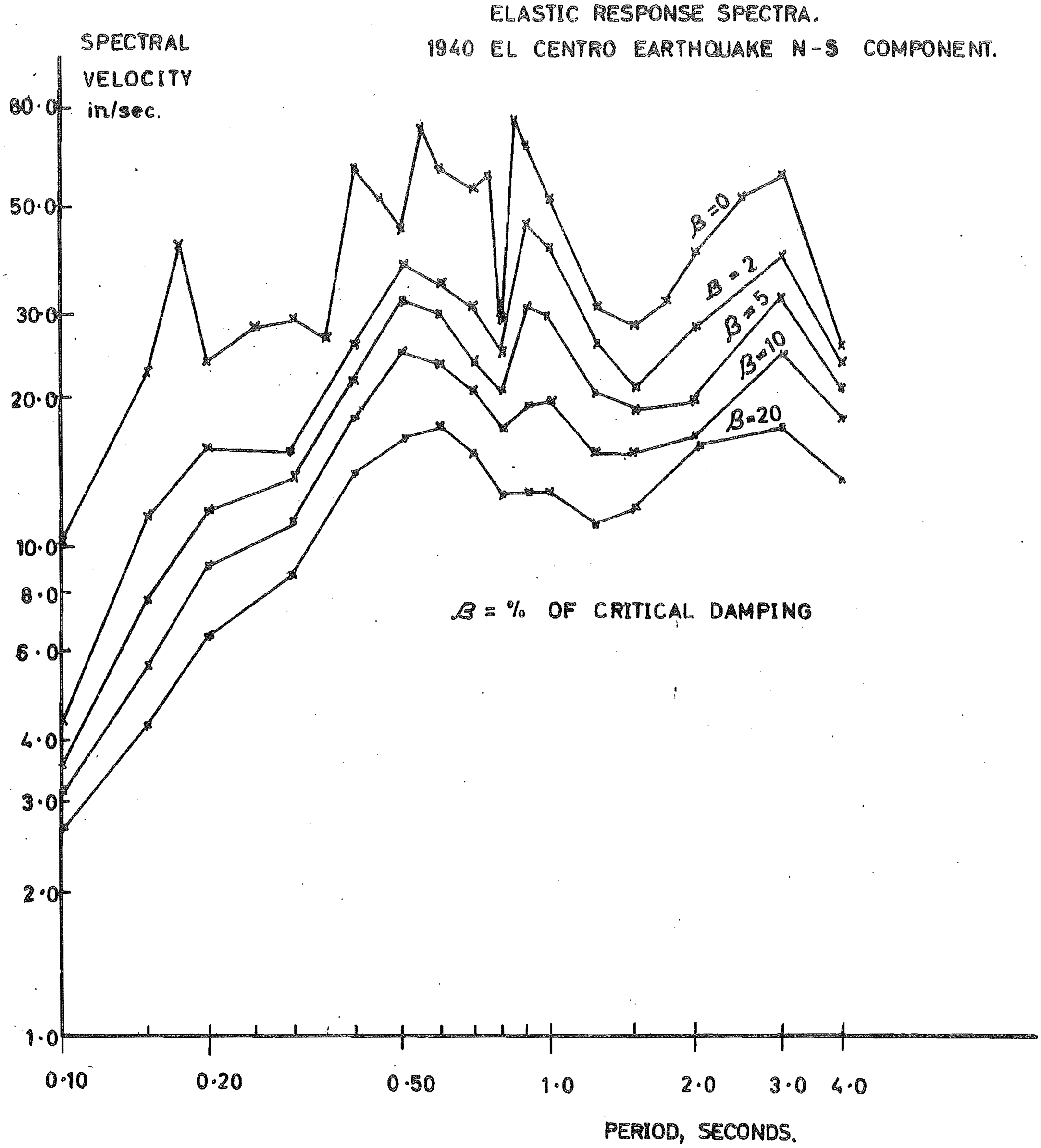

ELASTIC RESPONSE SPECTRA.
1940 EL CENTRO EARTHQUAKE N-S COMPONENT.
SPECTRAL VELOCITY in/sec.
60·0
50·0
30·0
20·0
10·0
8·0
6·0
4·0
3·0
2·0
1·0
0·10
0·20
0·50
1·0
2·0
3·0
4·0
PERIOD, SECONDS.
β=0
β=2
β=5
β=10
β=20
β = % OF CRITICAL DAMPING

FIGURE 3·4

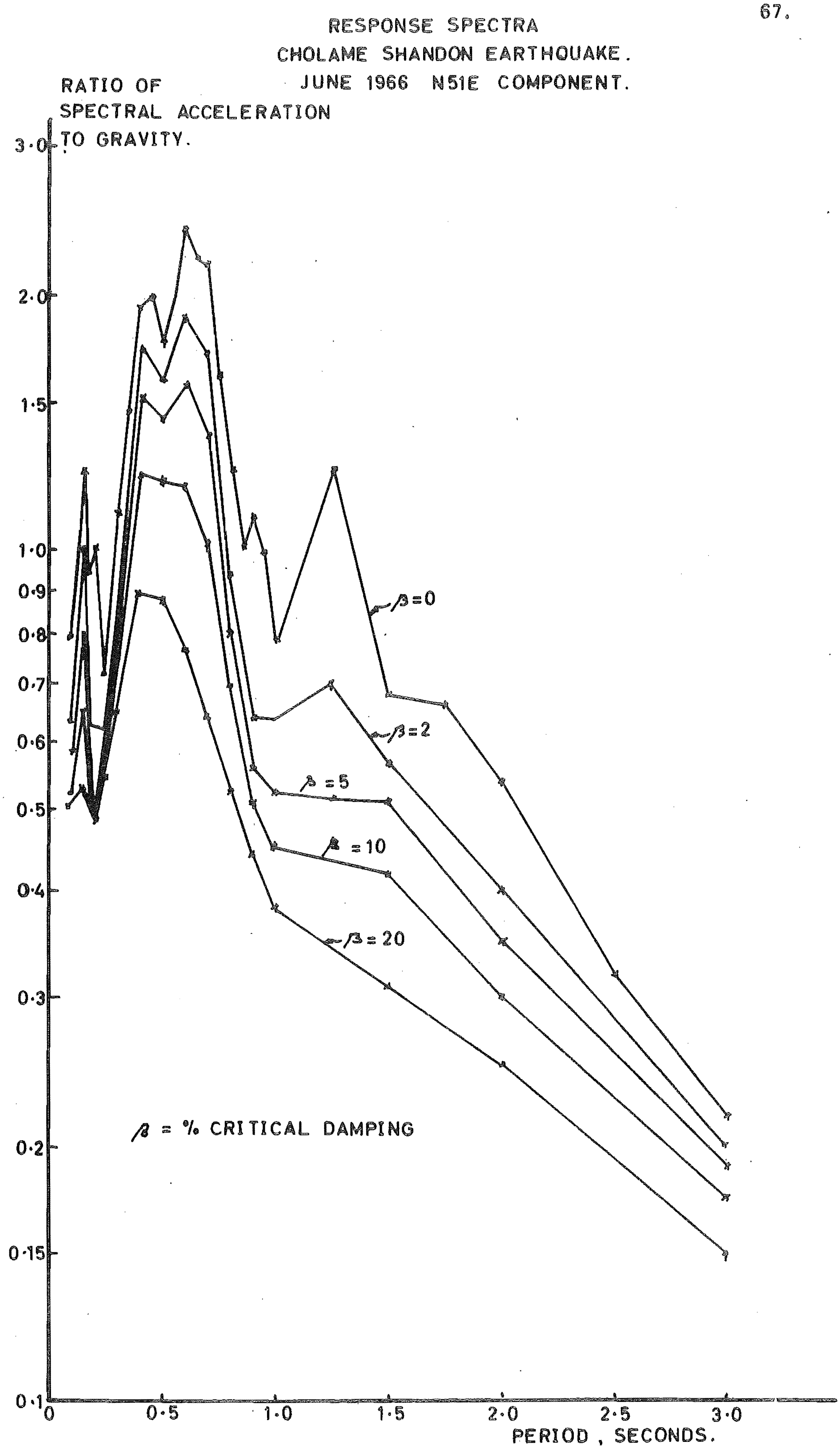

RESPONSE SPECTRA
CHOLAME SHANDON EARTHQUAKE.
JUNE 1966 N51E COMPONENT.
RATIO OF SPECTRAL ACCELERATION TO GRAVITY.
3·0
2·0
1·5
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·15
0·1
β=0
β=2
β=5
β=10
β=20
β = % CRITICAL DAMPING
0
0·5
1·0
1·5
2·0
2·5
3·0
PERIOD, SECONDS.

FIGURE 3·5

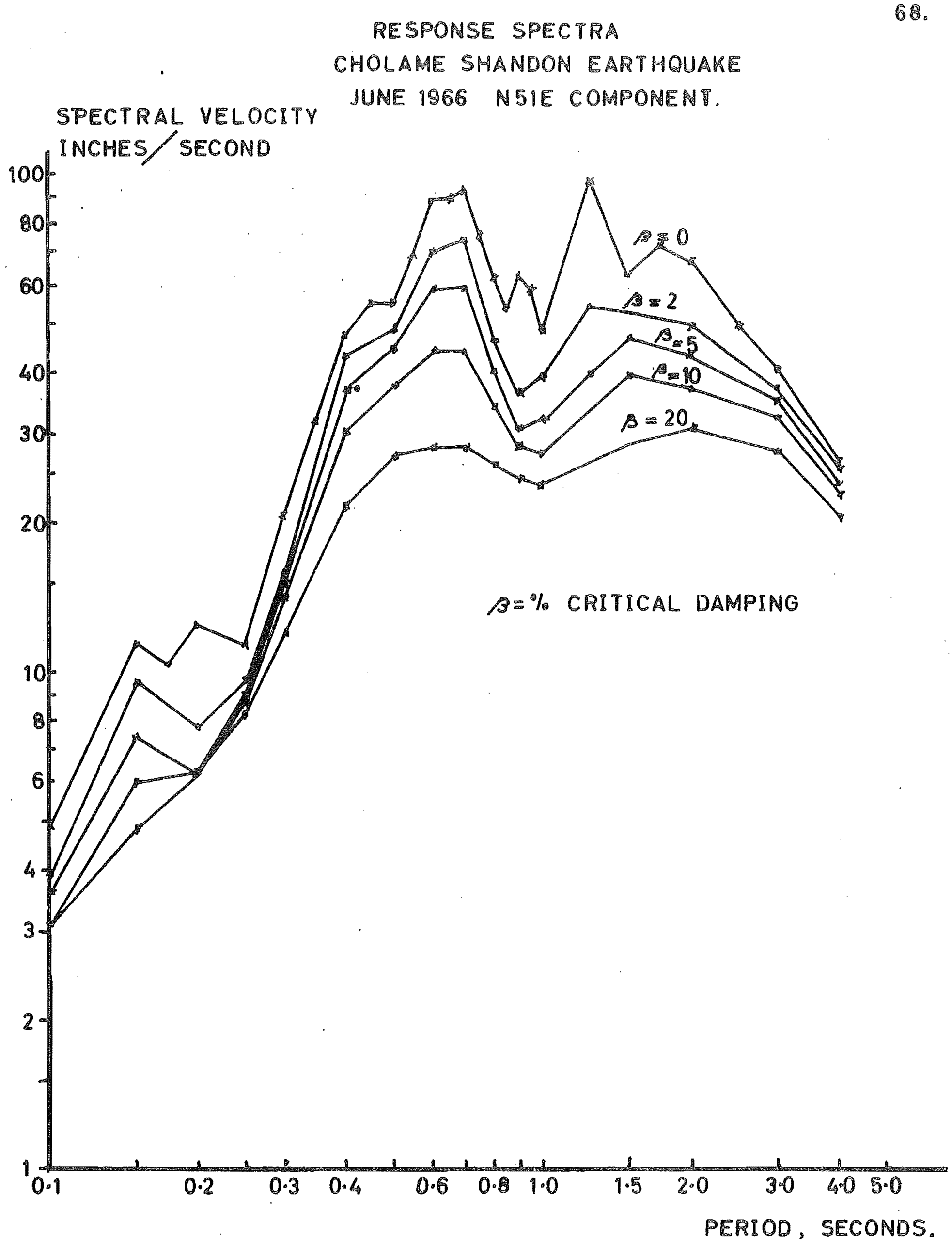
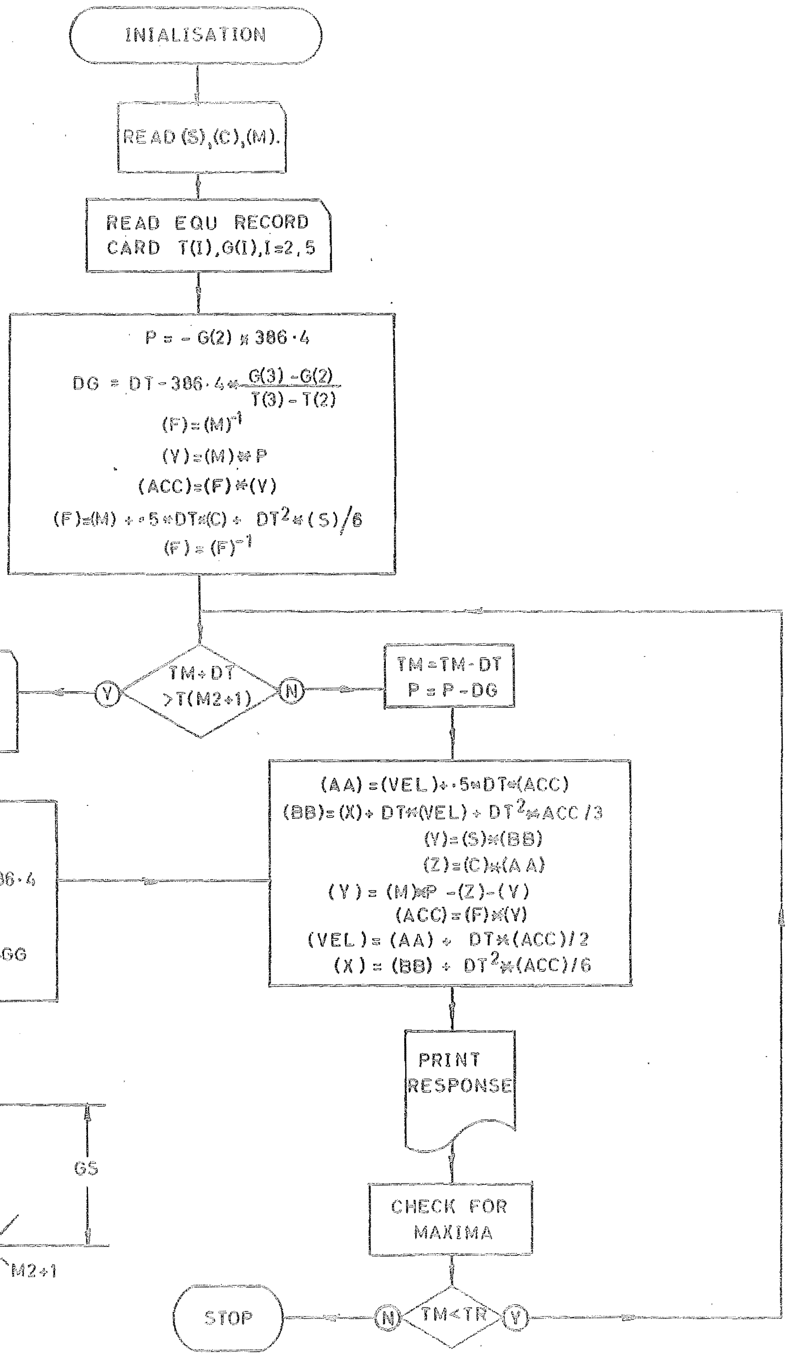
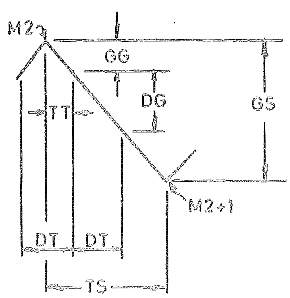

RESPONSE SPECTRA
CHOLAME SHANDON EARTHQUAKE
JUNE 1966 N51E COMPONENT.
SPECTRAL VELOCITY
INCHES/SECOND
100
80
60
40
30
20
10
8
6
4
3
2
1
β = 0
β = 2
β = 5
β = 10
β = 20
β = % CRITICAL DAMPING
0·1
0·2
0·3
0·4
0·6
0·8
1·0
1·5
2·0
3·0
4·0
5·0
PERIOD, SECONDS.

FIGURE 3·6

PROGRAM "ELRE5"
BLOCK DIAGRAM
FOR LIST 7





EARTHQUAKE RECORD.

FIGURE 3·7

```
C      PROGRAM ''ELRES''   LIST 7                                  PAGE 1

C      LANGUAGE - FORTRAN IV FOR IBM 360/44
C      THIS PROGRAM DETERMINES THE ELASTIC RESPONSE OF MULTI-MASS
C      SYSTEMS TO BERG FORMAT EARTHQUAKE RECORDS.
C      MS=1 ENTER (K)LAT,MS=2 ENTER (F)LAT
       DIMENSION S(20,20),F(20,20),QM(20),AA(20),BB(20),X(20),
      1VEL(20),ACC(20),Y(20),Z(20),T(5),G(5),BX(20),TX(20),C(20,20)
C      N=NO OF MASSES,KP=NO OF STEPS BEFORE PRINTING,DT=STEP INTERVAL
C      TR=LENGTH OF RECORD
    28 READ29,N,MS,KP,DT,TR
    29 FORMAT(3I3,2E10.4)
       PRINT32,N,MS,KP,DT,TR
    32 FORMAT(3I4,2E13.4)
       PRINT34
    34 FORMAT(//)
       KC=0
    30 FORMAT(8E10.4)
C      READ (K)LAT OR (F)LAT
       DO40I=1,N
       READ31,(S(I,J),J=1,N)
    40 PRINT143,(S(I,J),J=1,N)
   143 FORMAT(1X,10E13.4)
       PRINT34
C      READ DAMPING MATRIX
       DO41I=1,N
       READ31,(C(I,J),J=1,N)
    41 PRINT143,(C(I,J),J=1,N)
       PRINT34
    31 FORMAT(5E14.8)
       GO TO (2,3),MS
     3 CALL MATINV(S,N)
C      READ MASS MATRIX
     2 READ30,(QM(I),I=1,N)
       PRINT143,(QM(I),I=1,N)
       PRINT34
       DO5I=1,N
       X(I)=0.
       VEL(I)=0.
       BX(I)=0.
       QM(I)=QM(I)/386.4
       Z(I)=0.
       Y(I)=0.
       DO5J=1,N
     5 F(I,J)=0.
       IS=1
C      READ EQU. RECORD CARD
       READ33,ISC,(T(I),G(I),I=2,5)
       IF(IS-ISC)43,42,43
    33 FORMAT(I3,4(F8.4,F9.6))
    42 M2=2
       P=-G(2)*386.4
       DG=DT*386.4*(G(3)-G(2))/(T(3)-T(2))
       DO6I=1,N
     6 F(I,I)=1.0/QM(I)
       TM=0.
       DO7I=1,N
     7 Y(I)=QM(I)*P-Z(I)-Y(I)
       CALL VECMTP(ACC,F,Y,N,N)
       DO8I=1,N
       F(I,I)=QM(I)
       DO8J=1,N
     8 F(I,J)=F(I,J)&0.5*DT*C(I,J)&.16666667*DT*DT*S(I,J)
       CALL MATINV(F,N)
       DO 50 I=1,N
    50 PRINT143,(F(I,J),J=1,N)
       PRINT34
    25 IF(TM-TR&.5*DT)26,27,27
    26 IF(TM&DT-T(M2&1))10,10,11
    10 TM=TM&DT
       P=P-DG
       GO TO 12
    11 IF(M2-4)13,14,14
    13 M2=M2&1
       GO TO 15

C      PROGRAM ''ELRES''   LIST 7                                  PAGE 2
    14 M2=1
       G(1)=G(5)
       T(1)=T(5)
       IS=IS&1
       READ33,ISC,(T(I),G(I),I=2,5)
       IF(IS-ISC)43,15,43
    15 TT=TM&DT-T(M2)
       TS=T(M2&1)-T(M2)
       IF(TS)11,11,16
    16 GS=(G(M2&1)-G(M2))*386.4
       GG=GS*TT/TS
       DG=GS*DT/TS
       P=-G(M2)*386.4-GG
       TM=TM&DT
    12 DO17I=1,N
       AA(I)=VEL(I)&.5*DT*ACC(I)
    17 BB(I)=X(I)&DT*VEL(I)&.33333333*DT*DT*ACC(I)
       CALL VECMTP(Y,S,BB,N,N)
       CALL VECMTP(Z,C,AA,N,N)
       DO18I=1,N
    18 Y(I)=QM(I)*P-Z(I)-Y(I)
       CALL VECMTP(ACC,F,Y,N,N)
       DO19I=1,N
       VEL(I)=AA(I)&.5*DT*ACC(I)
    19 X(I)=BB(I)&.16666667*DT*DT*ACC(I)
       KC=KC&1
       IF(KC.LT.KP) GO TO 20
       PRINT143,TM,(X(I),I=1,N)
       KC=0
    20 DO22I=1,N
       IF(ABS(X(I))-ABS(BX(I)))22,22,23
    23 BX(I)=X(I)
       TX(I)=TM
    22 CONTINUE
       GO TO 25
    27 PRINT143,(BX(I),TX(I),I=1,N)
       STOP
    43 PRINT35,IS
    35 FORMAT(' CARDS OUT OF ORDER',I5)
       STOP 3333
       END
C
       SUBROUTINE MATINV(A,N)
C      DETERMINES INVERSE OF MATRIX A OF ORDER N.
       DIMENSION A(20,20)
       DO1I=1,N
       T=A(I,I)
       A(I,I)=1.0
       DO2J=1,N
     2 A(I,J)=A(I,J)/T
       DO1L=1,N
       IF(L-I)3,1,3
     3 T=A(L,I)
       A(L,I)=0.
       DO4J=1,N
     4 A(L,J)=A(L,J)-T*A(I,J)
     1 CONTINUE
       RETURN
       END
C
       SUBROUTINE VECMTP(B,A,Y,M,N)
C      (B)=(A)*(Y)  (A) OF ORDER MXN
       DIMENSION A(20,20),Y(20),B(20)
       DO1I=1,M
       T=0.
       DO2J=1,N
     2 T=T&A(I,J)*Y(J)
     1 B(I)=T
       RETURN
       END
```

LIST 7

```
C      PROGRAM ''SPECTRA''   LIST 8.                     PAGE 1
C      LANGUAGE - FORTRAN IV FOR IBM 360/44
C      THIS PROGRAM DETERMINES THE ELASTIC RESPONSE OF A SERIES OF
C      RESONATORS TO ENABLE A RESPONSE SPECTRUM TO BE PLOTTED.
       DIMENSION T(5),G(5),PE(100),D(100),S(100),C(100),X(100),VEL(100),
      1ACC(100),F(100),AA(100),BB(100),BX(100),BVEL(100),BACC(100),
      2OMEGA(100),TX(100)
       INP=5
       LNP=6
C      NR=NO OF RESONATORS,DT=STEP INTERVAL,TR=LENGTH OF RECORD.
    25 READ(INP,29) NR,DT,TR
    29 FORMAT(I3,2E10.4)
C      PE=PERIOD,D=PER CENT OF CRITICAL DAMPING
       READ(INP,30) (PE(I),D(I),I=1,NR)
    30 FORMAT(8E10.4)
       QM=1.0
       DO1I=1,NR
       X(I)=0.
       VEL(I)=0..
       BX(I)=0.
       BVEL(I)=0.
     1 BACC(I)=0.
       TM=0.
       WRITE(LNP,39) NR,DT,TR
    39 FORMAT(1X,I3,2E13.4//)
       DO19I=1,NR
       S(I)=6.2831853/PE(I)
       OMEGA(I)=S(I)
       S(I)=S(I)*S(I)*QM
    19 C(I)=D(I)*0.02*SQRT(S(I)*QM)
       IS=1
       READ(INP,31) ISC,(T(I),G(I),I=2,5)
    31 FORMAT(I3,4(F8.4,F9.6))
       IF(IS-ISC)50,51,50
    51 M2=2
       P=-G(2)*386.4
       DG=DT*386.4*(G(3)-G(2))/(T(3)-T(2))
       DO20I=1,NR
       ACC(I)=P-(C(I)*VEL(I)&S(I)*X(I))
    20 F(I)=1.0/(QM&0.5*DT*C(I)&.16666667*DT*DT*S(I))
     2 IF(TM-TR&0.5*DT)21,22,22
    21 IF(TM&DT-T(M2&1))3,3,4
     3 TM=TM&DT
       P=P-DG
       GO TO 5
     4 IF(M2-4)6,7,7
     6 M2=M2&1
       GO TO 8
     7 M2=1
       G(1)=G(5)
       T(1)=T(5)
       IS=IS&1
       READ(INP,31) ISC,(T(I),G(I),I=2,5)
       IF(IS-ISC)50,8,50
    50 WRITE(LNP,45) IS
    45 FORMAT(1X,21HCARD OUT OF ORDER NO.I3)
       STOP
     8 TT=TM&DT-T(M2)
       TS=T(M2&1)-T(M2)
       IF(TS)4,4,18
    18 GS=(G(M2&1)-G(M2))*386.4
```

```
C      PROGRAM ''SPECTRA''   LIST 8.                     PAGE 2
       GG=GS*TT/TS
       DG=GS*DT/TS
       P=-G(M2)*386.4-GG
       TM=TM&DT
     5 DO15I=1,NR
       AA(I)=VEL(I)&0.5*DT*ACC(I)
       BB(I)=X(I)&DT*VEL(I)&.33333333*DT*DT*ACC(I)
       ACC(I)=(QM*P-C(I)*AA(I)-S(I)*BB(I))*F(I)
       VEL(I)=AA(I)&0.5*DT*ACC(I)
       X(I)=BB(I)&.16666667*DT*DT*ACC(I)
       IF(ABS(X(I))-ABS(BX(I)))11,11,12
    12 BX(I)=X(I)
       TX(I)=TM
    11 IF(ABS(VEL(I))-ABS(BVEL(I)))13,13,14
    14 BVEL(I)=VEL(I)
    13 IF(ABS(ACC(I))-ABS(BACC(I)))15,15,16
    16 BACC(I)=ACC(I)
    15 CONTINUE
       GO TO 2
    22 DO23I=1,NR
       BACC(I)=BACC(I)/386.4
       P=OMEGA(I)*BX(I)
       GG=P*OMEGA(I)/386.4
C      BX=MAX DISPLACEMENT,P=SPECTRAL VELOCITY,GG=SPECTRAL ACCELERATION,
C      TX=TIME OF MAX. DISPL., BVEL=MAX. VEL., BACC=MAX. ACC.
    23 WRITE(LNP,40) PE(I),D(I),BX(I),P,GG,TX(I),BVEL(I),BACC(I)
    40 FORMAT(/1X,8E13.4)
       WRITE(LNP,41)
    41 FORMAT(///,1X,19HPROCESSING COMPLETE)
       STOP
       END
```

LIST 8

```
C     PROGRAM ''SUMMOD''    LIST 9.                        PAGE 1
C     LANGUAGE - FORTRAN PDQ FOR IBM 1620.
C
C     THIS PROGRAM DETERMINES THE MAXIMUM ELASTIC RESPONSE OF A
C     MULTI-MASS SYSTEM BY SUMMING THE RESPONSE OF ITS NORMAL
C     MODES. ACCEPTS BERG FORMAT EQU. RECORDS.
      DIMENSION T(5),G(5),S(20),C(20),X(20),VEL(20),ACC(20),F(20)
      DIMENSION AA(20),BB(20),BX(20),TX(20),OMEGA(20),ONEG(20,6),XS(20)
C     NR=NO OF MODES, NS=NO OF MASSES,
C     DT=STEP INTERVAL, TR=LENGTH OF EQU. RECORD.
   25 READ29,NR,NS,DT,TR
      PRINT29,NR,NS,DT,TR
   29 FORMAT(2I3,2E10.4)
      DO26I=1,NR,8
C     S=PERIOD OF MODE
      READ30,S(I),S(I&1),S(I&2),S(I&3),S(I&4),S(I&5),S(I&6),S(I&7)
   26 PRINT30,S(I),S(I&1),S(I&2),S(I&3),S(I&4),S(I&5),S(I&6),S(I&7)
   30 FORMAT(8E10.4)
      DO27I=1,NR,8
C     C=PER CENT OF CRITICAL DAMPING IN MODE
      READ30,C(I),C(I&1),C(I&2),C(I&3),C(I&4),C(I&5),C(I&6),C(I&7)
   27 PRINT30,C(I),C(I&1),C(I&2),C(I&3),C(I&4),C(I&5),C(I&6),C(I&7)
C     ONEG=ONE G DISPLACEMENTS.
      DO50I=1,NR
      DO50J=1,NS,8
      READ30,ONEG(J,I),ONEG(J&1,I),ONEG(J&2,I),ONEG(J&3,I),
     1ONEG(J&4,I),ONEG(J&5,I),ONEG(J&6,I),ONEG(J&7,I)
   50 PRINT30,ONEG(J,I),ONEG(J&1,I),ONEG(J&2,I),ONEG(J&3,I),
     1ONEG(J&4,I),ONEG(J&5,I),ONEG(J&6,I),ONEG(J&7,I)
      QM=1.0
      DO1I=1,NR
      X(I)=0.
    1 VEL(I)=0.
      N=(NS&3)/4*4
      DO14I=1,N
      BX(I)=0.
   14 TX(I)=0.
      TM=0.
      DO19I=1,NR
      OMEGA(I)=6.2831853/S(I)
      OMEGA(I)=OMEGA(I)*OMEGA(I)
      S(I)=OMEGA(I)*QM
   19 C(I)=C(I)*0.02*SQRT(S(I)*QM)
      IS=1
C     READ EQU. RECORD CARD
      READ31,ISC,T(2),G(2),T(3),G(3),T(4),G(4),T(5),G(5)
   31 FORMAT(I3,F8.4,F9.6,F8.4,F9.6,F8.4,F9.6,F8.4,F9.6)
      IF(IS-ISC)40,41,40
   41 M2=2
      P=-G(2)*386.4
      DG=DT*386.4*(G(3)-G(2))/(T(3)-T(2))
      DO20I=1,NR
      ACC(I)=P-(C(I)*VEL(I)&S(I)*X(I))/QM
   20 F(I)=1.0/(QM&0.5*DT*C(I)&.16666667*DT*DT*S(I))
    2 IF(SENSE SWITCH 1)22,61
   61 IF(TM-TR)21,22,22
   21 IF(TM&DT-T(M2&1))3,3,4
    3 TM=TM&DT
      P=P-DG
      GO TO 5
    4 IF(M2-4)6,7,7
```

```
C     PROGRAM ''SUMMOD''    LIST 9.                        PAGE 2
    6 M2=M2&1
      GO TO 43
    7 M2=1
      T(1)=T(5)
      G(1)=G(5)
      IS=IS&1
C     READ EQU. RECORD CARD
   42 READ31,ISC,T(2),G(2),T(3),G(3),T(4),G(4),T(5),G(5)
      IF(IS-ISC)40,43,40
   43 TT=TM&DT-T(M2)
      TS=T(M2&1)-T(M2)
      IF(TS)4,4,18
   18 GS=(G(M2&1)-G(M2))*386.4
      GG=GS*TT/TS
      DG=GS*DT/TS
      P=-G(M2)*386.4-GG
      TM=TM&DT
    5 DO15I=1,NR
      AA(I)=VEL(I)&0.5*DT*ACC(I)
      BB(I)=X(I)&DT*VEL(I)&.33333333*DT*DT*ACC(I)
      ACC(I)=(QM*P-C(I)*AA(I)-S(I)*BB(I))*F(I)
      VEL(I)=AA(I)&0.5*DT*ACC(I)
   15 X(I)=BB(I)&.16666667*DT*DT*ACC(I)
      DO9I=1,NS
      SUM=0.
      DO8J=1;NR
    8 SUM=SUM&ONEG(I,J)*X(J)*OMEGA(J)
      XS(I)=SUM
      IF(ABS(XS(I))-ABS(BX(I)))9,9,12
   12 BX(I)=XS(I)
      TX(I)=TM
    9 CONTINUE
      GO TO 2
   22 DO11I=1,NS
   11 BX(I)=BX(I)/386.4
      DO13I=1,NS,4
C     BX=MAX FLOOR DISPL., TX=TIME OF BX.
   13 PRINT30,BX(I),TX(I),BX(I&1),TX(I&1),BX(I&2),TX(I&2),
     1BX(I&3),TX(I&3)
      PAUSE
      GO TO 25
   40 PRINT39,IS
   39 FORMAT(19HCARDS OUT OF ORDER.I5)
      PAUSE
      GO TO 42
      END
```

CHAPTER FOUR

DYNAMIC ELASTO-PLASTIC RESPONSE

4.1 INTRODUCTION

The determination of the response of multi-storey buildings to a recorded earthquake ground motion, assuming the frame remains completely elastic, has been described in Chapter 3. The elastic response may be estimated using the normal mode-response spectrum approach, or be determined by integrating either the independent modal equations of motion or the equations of motion of the floor masses.

When the elastic response of a typical multi-storey building to a major earthquake is calculated, it is clear that the stresses in some of the members are greater than the yield stress of the material. This is because many framed structures are designed on the basis that they will resist the more frequent moderate ground motions without damage, but will withstand the most intense seismic shocks without total collapse occurring only by calling on the reserve strength beyond the yield deformation point in the individual members.

It has been common practice to design structures to yield when resisting earthquake forces of the order of one quarter of those predicted by an elastic response analysis. Studies[8] of one degree of freedom elasto-plastic systems subjected to earthquake motions showed that the maximum displacements were reasonably independent of the yield strength of the structures, and these studies justified this design practice to some extent because a reduction factor of one quarter implied a ductility factor of 4 for a true one degree of freedom elasto-plastic system and it was considered that most structural components would be capable of providing a ductility factor of 4 without fracture.

However, these studies actually only justified the ductility reduction factor concept for structures which responded as true one degree of freedom elasto-plastic systems, although the results could be applied qualitatively to more complex systems.

Even in very simple structures plastic deformations are seldom distributed similarly to the elastic deformations. Yielding may be expected to destroy the elastic mode vibration characteristics which form the basis of the mode superposition techniques so it would not be possible to accurately predict the response of a yielding multi-degree of freedom system using response spectrum techniques combined with the ductility factor concept.

In practice yielding may be an extremely localised phenomenon or it may be widely distributed and the magnitudes of the required ductilities are not necessarily equal to the factor used to reduce the elastic response forces to the design yield forces. If the yielding of a complex system occurs only at a few points it would be expected that the ductility required there would greatly exceed the reduction factor.

In the past the absorption of energy by inelastic deformation of the frame was not of such critical importance because in the older style of buildings non-structural partitions and walls had a large capacity for energy absorption, but in the modern style of high rise building with light-weight partition walls and thin cladding, both of which may be of negligible strength, the structural frame must provide all the inelastic energy absorption.

As there is some doubt about whether the structural components in use are capable of providing ductile deformations of large magnitude, it is desirable to know the ductilities required in the members of a typical earthquake-resistant structure.

At the beginning of this chapter the methods of determining the elastic response of structures to earthquakes were mentioned. Only the last of these, the numerical integration of the equations of motion of the masses, may be conveniently extended to determine the elasto-plastic response of a structure, because yielding destroys the normal mode properties.

4.2 DEVELOPMENT OF THE NUMERICAL INTEGRATION METHOD

The elasto-plastic analysis is achieved by the step-by-step numerical integration of the differential equations of motion and within each short step interval of time the structure is assumed to behave in a linear elastic manner, as before, but the elastic properties of the structure are changed from one interval to another as dictated by the response, so that the non-linear response is obtained as a sequence of the linear responses of different systems. For each successive time interval the stiffness of the frame is evaluated based on the moments in the members at the beginning of the increment, and the changes in deformation of the linear system are computed by integrating the differential equations of motion over the finite step interval. The total deformations are found by adding the incremental deformations, which are also used to calculate the increase in member end actions from the member stiffness relationships. Stiffness coefficients for the next time interval may be found according to the yield conditions of the member.

In matrix form the dynamic equilibrium of a multi-storey frame rigidly fixed to the ground with distributed mass lumped at the floor levels, and subjected to excitation of the base, may be expressed as follows:

$$\begin{bmatrix} \ddots & \\ & [M] \end{bmatrix} \{\ddot{X}\} + \begin{bmatrix} \ddots & \\ & [C] \end{bmatrix} \{\dot{X}\} + \begin{bmatrix} & K & \end{bmatrix} \begin{Bmatrix} \{\theta\} \\ \{X\} \end{Bmatrix} = -\ddot{X}_g \begin{bmatrix} \ddots & \\ & [M] \end{bmatrix} \qquad (1)$$

$\{\ddot{X}\}$ = Acceleration vector of the floors relative to the base

$\{\dot{X}\}$ = Velocity " " " " " " " "

$\{X\}$ = Displacement " " " " " " " "

$\{\theta\}$ = Rotation vector of the joints of the frame

$[M]$ = Diagonal matrix of the storey masses

$[C]$ = Damping matrix

$[K]$ = Stiffness matrix for the frame

$\ddot{X}g$ = Acceleration of the ground.

This system of equations is actually non-linear because the stiffness matrix $[K]$ is dependent on the magnitude of the response. However it is assumed that the structural system remains linear over a very short time increment, Δt.

Then

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & [M] \end{bmatrix} \begin{Bmatrix} \cdot \\ \{\Delta \ddot{x}\} \end{Bmatrix} + \begin{bmatrix} \cdot & \cdot \\ \cdot & [C] \end{bmatrix} \begin{Bmatrix} \cdot \\ \{\Delta \dot{x}\} \end{Bmatrix} + \begin{bmatrix} & K & \end{bmatrix} \begin{Bmatrix} \{\Delta \theta\} \\ \{\Delta x\} \end{Bmatrix} = -\Delta \ddot{x}_g \begin{bmatrix} \cdot & \cdot \\ \cdot & [M] \end{bmatrix} \quad (2)$$

A method of numerical integration may be derived from the Newmark β method[9] which assumes a linear variation of acceleration when $\beta = \frac{1}{6}$

Then

$$\{\dot{X}\}_t = \{\dot{X}\}_{t-\Delta t} + \frac{1}{2}\Delta t \{\ddot{X}\}_{t-\Delta t} + \frac{1}{2}\Delta t \{\ddot{X}\}_t$$

$$\{X\}_t = \{X\}_{t-\Delta t} + \Delta t \{\dot{X}\}_{t-\Delta t} + \frac{1}{3}\Delta t^2 \{\ddot{X}\}_{t-\Delta t} + \frac{1}{6}\Delta t^2 \{\ddot{X}\}$$

where subscript t denotes the response at time t and $(t-\Delta t)$ that at time $(t-\Delta t)$.

It may be derived from these two equations that

$$\{\Delta \dot{X}\} = \frac{3}{\Delta t}\{\Delta X\} + \{B\}$$

where $\{B\} = -3\{\dot{X}\}_{t-\Delta t} - \frac{\Delta t}{2}\{\ddot{X}\}_{t-\Delta t}$

and $\{\Delta \ddot{X}\} = \frac{6}{\Delta t^2}\{\Delta X\} + \{A\}$

where $\{A\} = -\frac{6}{\Delta t}\{\dot{X}\}_{t-\Delta t} - 3\{\ddot{X}\}_{t-\Delta t}$

These expressions for $\{\Delta\dot{X}\}$ and $\{\Delta\ddot{X}\}$ may be substituted into the equation (2) to give a matrix equation which may be solved for $\{\Delta X\}$

$$[K^*]\{\Delta y\} = \{\Delta R\}$$

where

$$[K^*] = \frac{6}{\Delta t^2}\begin{bmatrix} 0 & 0 \\ 0 & [M] \end{bmatrix} + \frac{3}{\Delta t}\begin{bmatrix} 0 & 0 \\ 0 & [C] \end{bmatrix} + \begin{bmatrix} K \end{bmatrix}$$

$$\{\Delta R\} = \left\{-\Delta\ddot{X}_g\begin{bmatrix} 0 & 0 \\ 0 & [M] \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & [M] \end{bmatrix}\begin{Bmatrix} 0 \\ \{A\} \end{Bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & [C] \end{bmatrix}\begin{Bmatrix} 0 \\ \{B\} \end{Bmatrix}\right\}$$

$$\{\Delta y\} = \begin{Bmatrix} \Delta\theta \\ \cdots \\ \Delta X \end{Bmatrix}$$

The numerical process consists of initialising the values of the relative velocity and displacement vectors to zero, assuming that the building is initially at rest. Then the terms of the relative acceleration vector are found to be equal to the initial acceleration of the ground from equation (1)

i.e. $\{\ddot{X}\}_0 = -\ddot{X}g\ \{1\}$

The following steps are then executed repeatedly:

$$\{A\} = -\frac{6}{\Delta t}\{\dot{X}\} - 3\{\ddot{X}\}$$

$$\{B\} = -3\{\dot{X}\} - \frac{\Delta t}{2}\{\ddot{X}\}$$

$$\{\Delta R\} = -\Delta\ddot{X}g\,.\left[\ddots\;M\right] - \left[\ddots\;M\right]\{A\} - \left[\ddots\;C\right]\{B\}$$

$$\left[K^{*}\right] = \left[\left[K\right] + \frac{3}{\Delta t}\cdot\left[\ddots\;C\right] + \frac{6}{\Delta t^{2}}\cdot\left[\ddots\;M\right]\right]$$

$$\{\Delta y\} = \left[K^{*}\right]^{-1}\{\Delta R\}$$

$$\{X\} = \{X\} + \{\Delta X\}$$

$$\{\dot{X}\} = \{\dot{X}\} + \frac{3}{\Delta t}\{X\} + \{B\}$$

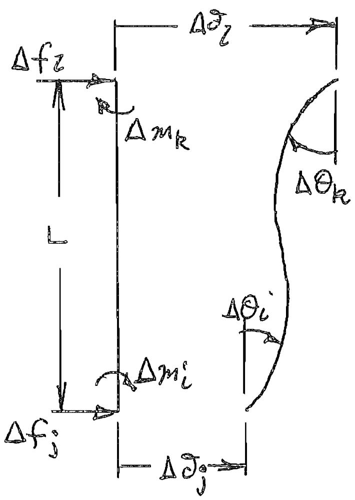
$$\{\ddot{X}\} = \{\ddot{X}\} + \frac{6}{\Delta t^{2}}\{X\} + \{A\}$$

The inverse of $\left[K^{*}\right]$ is found on the first pass and is used repeatedly in the following steps until the pattern of plastic hinges changes. The matrix $\left[K^{*}\right]$ is then set up again by assembling the stiffness matrix for the frame, from the member stiffness matrices, allowing for the plastic hinges present and adding the effects of the damping and mass matrices.

4.3 MEMBER STIFFNESS MATRICES

If bending deformations only are considered, the incremental moment-deformation relationships for a column member may be written as follows:

1. With no hinges present :



$$\begin{Bmatrix} \Delta m_i \\ \Delta f_j \\ \Delta m_k \\ \Delta f_l \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 4 & \frac{6}{L} & 2 & -\frac{6}{L} \\ \frac{6}{L} & \frac{12}{L^2} & \frac{6}{L} & -\frac{12}{L^2} \\ 2 & \frac{6}{L} & 4 & -\frac{6}{L} \\ -\frac{6}{L} & -\frac{12}{L^2} & -\frac{6}{L} & +\frac{12}{L^2} \end{bmatrix} \begin{Bmatrix} \Delta \theta_i \\ \Delta \delta_j \\ \Delta \theta_k \\ \Delta \delta_l \end{Bmatrix}$$

where Δm denotes the increase in an action which is a bending moment,

Δf denotes the increase in an action which is a force,

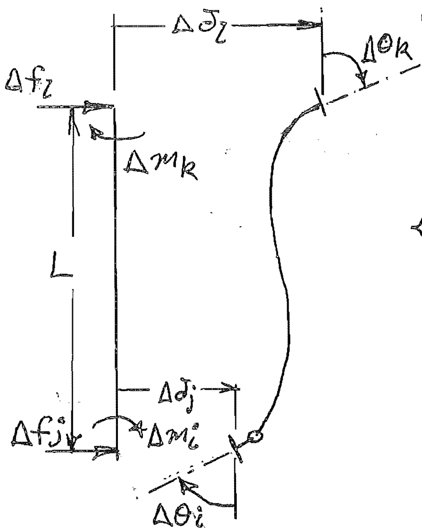
$\Delta \theta$ denotes the increase in a frame deformation which is a rotation,

$\Delta \delta$ denotes the increase in a frame deformation which is a displacement.

Subscripts are used to distinguish individual actions and deformations.

E, I and L are as previously defined.

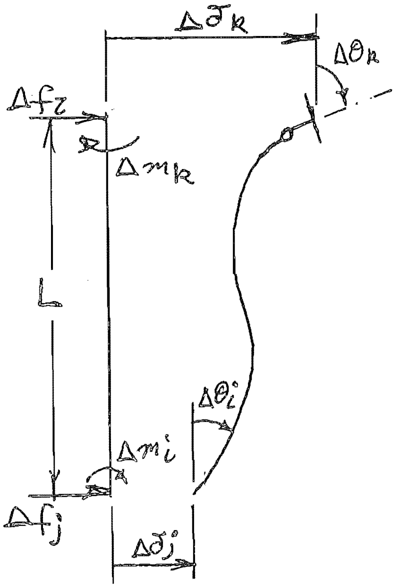
2. With a hinge at i



$$\begin{Bmatrix} \Delta m_i \\ \Delta f_j \\ \Delta m_k \\ \Delta f_l \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{3}{L^2} & \frac{3}{L} & -\frac{3}{L^2} \\ 0 & \frac{3}{L} & 3 & -\frac{3}{L} \\ 0 & -\frac{3}{L^2} & -\frac{3}{L} & \frac{3}{L^2} \end{bmatrix} \begin{Bmatrix} \Delta \theta_i \\ \Delta \delta_j \\ \Delta \theta_k \\ \Delta \delta_l \end{Bmatrix}$$

It should be noted that $\Delta\theta_i$ is the increase in rotation of joint i of the frame and not the increase in rotation of end i of the member under consideration. The rotation of the plastic hinge is given by the difference between $\Delta\theta_i$ and the increase in rotation of end i of the member.

3. With a hinge at k



$$\begin{Bmatrix} \Delta m_i \\ \Delta f_j \\ \Delta m_k \\ \Delta f_l \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 3 & \frac{3}{L} & 0 & -\frac{3}{L} \\ \frac{3}{L} & \frac{3}{L^2} & 0 & -\frac{3}{L^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{3}{L^2} & 0 & +\frac{3}{L^2} \end{bmatrix} \begin{Bmatrix} \Delta \theta_i \\ \Delta \delta_j \\ \Delta \theta_k \\ \Delta \delta_l \end{Bmatrix}$$

4. With hinges at i and k the stiffness matrix becomes null.

$$\begin{Bmatrix} \Delta m_i \\ \Delta f_j \\ \Delta m_k \\ \Delta f_l \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta \theta_i \\ \Delta \delta_j \\ \Delta \theta_k \\ \Delta \delta_l \end{Bmatrix}$$

With a beam member the displacement deformations are completely restrained, and the equilibrium equations are as follows:

1. With no hinges

$$\begin{Bmatrix} \Delta m_i \\ \Delta m_j \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} \Delta \theta_i \\ \Delta \theta_j \end{Bmatrix}$$

2. With a hinge at i

$$\begin{Bmatrix} \Delta m_i \\ \Delta m_j \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \begin{Bmatrix} \Delta \theta_i \\ \Delta \theta_j \end{Bmatrix}$$

3. With a hinge at j

$$\begin{Bmatrix} \Delta m_i \\ \Delta m_j \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\theta_i \\ \Delta\theta_j \end{Bmatrix}$$

4. With hinges at i and j

$$\begin{Bmatrix} m_i \\ m_j \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\theta_i \\ \Delta\theta_j \end{Bmatrix}$$

4.4 ASSEMBLY OF THE FRAME STIFFNESS MATRIX

The frame stiffness matrix is assembled by adding the member stiffness matrices. When the member data is read, the numbers of the four end deformations associated with the member are read and this enables the program to calculate the member stiffness coefficients and add them into the appropriate position in the frame stiffness matrix. If any of the four end deformations are restrained, as at the base of the bottom storey columns, this is indicated by assigning a number greater than the number of frame joint deformations and the program does not set up the member stiffness coefficients associated with that deformation.

The member stiffness matrices are written in terms of frame deformations and so it is not necessary to make any co-ordinate transformations.

4.5 MOMENT ROTATION CHARACTERISTICS

At the end of each step interval it is necessary to calculate the end moments of every member to check if a plastic hinge has been formed. This is done by utilising the incremental moment - deformation stiffness relationship. The program also checks to see if the plastic deformation associated with a hinge is compatible with the sign of the moment. The plastic hinge is free to rotate in one direction only, and in the other direction the section becomes elastic. The assumed moment-rotation characteristics of the members are of the type illustrated in Figure 4.1.

The conditions that the program imposes are:

1. The moment cannot exceed the ultimate moment.

2. If a moment is less than ultimate, the hinge cannot rotate.

3. If the moment is equal to the ultimate value, then the hinge may rotate in a direction consistent with the sign of the moment.

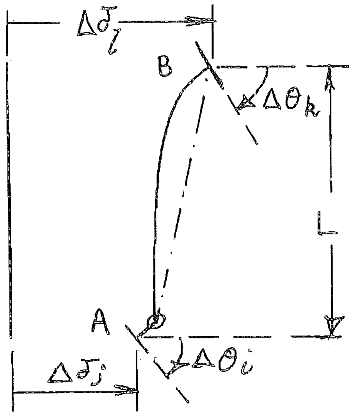
4. If the hinge starts to rotate in a direction inconsistent with the sign of the moment, the hinge is removed.

4.6 CALCULATION OF THE PLASTIC DEFORMATIONS

The incremental rotations of the plastic hinges of all the members are calculated from the increases in the joint deformations of the frame.

The following relationships may be derived by imposing each joint deformation in turn and applying moment area principles, considering bending deformations only.

With a hinge at end A only, the increase in rotation of the hinge Δp_A is given by:

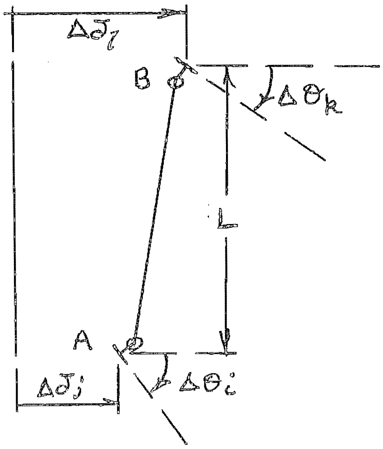


$$\Delta p_A = \Delta\theta_i + \tfrac{1}{2}\Delta\theta_k + 1{\cdot}5\,\frac{\Delta\delta_j - \Delta\delta_l}{L}$$

With a hinge at end B only, the increase in rotation of the hinge Δp_B is given by:

$$\Delta p_B = \Delta\theta_k + \tfrac{1}{2}\Delta\theta_i + 1{\cdot}5\,\frac{\Delta\delta_j - \Delta\delta_l}{L}$$

With hinges at A and B:



$$\Delta p_A = \Delta\theta_i + \frac{\Delta\delta_j - \Delta\delta_l}{L}$$

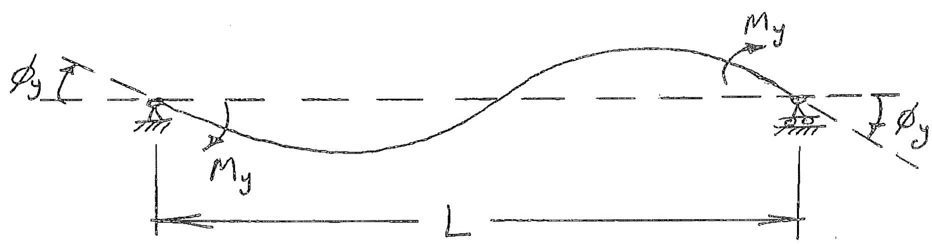
$$\Delta p_B = \Delta\theta_k + \frac{\Delta\delta_j - \Delta\delta_l}{L}$$

4.7 EARTHQUAKE RECORDS

The program has been written to accept the punched card accelerograms prepared by Professor G.V. Berg of Michigan University. These represent the earthquake as a piecewise linear function, the co-ordinates, i.e. time and acceleration, of the peaks and troughs being punched on cards with four points per card. Each card has a sequence number and the program checks to see if the cards are in order. These records have been corrected[22] so that the integrated velocity and displacement of the ground have reasonable values. The El Centro 1940 N-S component record was used to excite all frames for which an elasto-plastic analysis was carried out.

4.8 CALCULATION OF MEMBER DUCTILITY RATIO

The non-linear column and beam deformations are expressed in terms of the member ductility ratio. This is defined as the ratio of maximum total end rotation in the member to the end rotation at the elastic limit. The elastic limit rotation is the angle developed when the member is subjected to anti-symmetric yield moments M_y.



Then $\phi_y = \frac{M_y L}{6EI}$

The ductility factor μ is then defined as

$$\mu = \frac{\phi_y + \beta_{max}}{\phi_y}$$

$$= 1 + \frac{\beta_{max}}{\phi_y}$$

$$= 1 + \frac{6EI}{M_y L} \cdot \beta_{max}$$

where β_{max} is the maximum rotation of the plastic hinge.

4.9 STEP INTERVAL

For this linear acceleration numerical integration procedure Newmark[9] has suggested that the time increment Δt should be $\frac{1}{6}$ to $\frac{1}{10}$ of the smallest period of the structure. The nth period of an n storey "shear" building is given by the formula:

$$T_n = \frac{T_1}{2n - 1}$$

where T_1 is the fundamental period.

For a 20 storey "shear" building:

$$T_{20} = .026\ T_1$$

$$\Delta t \leqslant \frac{1}{6}\ T_{20}$$

$$\leqslant .004\ T_1$$

If $T_1 = 0.5$ secs

$$\Delta t \leqslant .002 \text{ secs.}$$

The other criterion which the time interval must satisfy is that the earthquake record must be adequately represented. The record is accepted as a piecewise linear function and the co-ordinates of the peaks and troughs given in digital form. The program interpolates between the co-ordinate points to obtain the acceleration of the ground at the beginning of each step interval, and clearly the time increment must be small compared with the time between co-ordinate points to give adequate representation of the exciting function. By trial it was found that a step interval not greater than .02 secs. gave a satisfactory result. This requirement is not as severe as the one above.

The other reason for keeping the step interval fairly small is that the structure is assumed to remain linear within each step interval and the yield conditions of the members are not checked until the end of the step interval. If a member has reached yield or rotated in a direction incompatible with the plastic hinge, then a plastic hinge is inserted or removed for the next time increment, but the previous time increment is not re-iterated to find exactly when the change in yield condition occurred. This process obviously introduces some error, but provided the step interval is sufficiently small these errors would not be significant.

There is no direct way of knowing when the step interval is sufficiently small, and the simplest procedure is to keep reducing the time increment until the same solution is obtained. Unfortunately while the time increment is being reduced the effect of the truncation errors increases, because the response is obtained as the sum of the incremental responses and it was found with the IBM 360/44 computer that when the step interval was reduced below .005 secs the solution became obviously unstable. This difficulty was overcome by storing the critical variables as double precision words and it was then found that a stable solution was obtained as the step interval was reduced. A step interval $\frac{1}{40}$ of the fundamental period was found to give a satisfactory result for the frames studied.

4.10 THE COMPUTER PROGRAM "DYNEPRES"

A computer program "DYNEPRES" has been written using the principles outlined above and a block diagram is given in Figure 4.2 and the listing in List 10. The program has been dimensioned to analyse up to a 14 storey by $1\frac{1}{2}$ bay frame.

The program has been used to assist with the dynamic analysis of various frames, and the results of these analyses are given in the next chapter. Some modifications which were needed to take account of joint size and axial deformation are described in Chapter 6. The programs which were described in Chapters 2 and 3 have been used to carry out elastic analyses for comparative purposes.

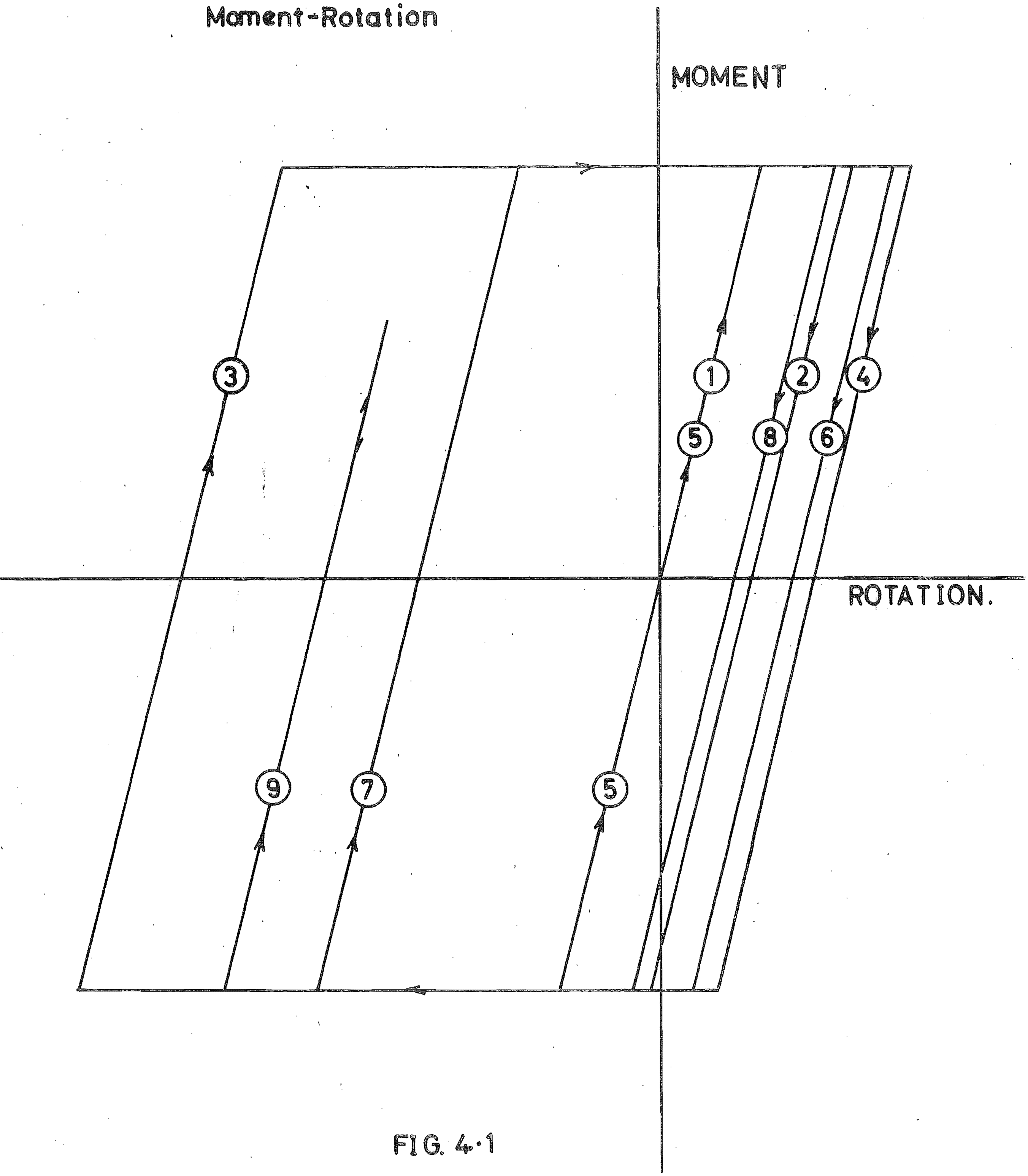
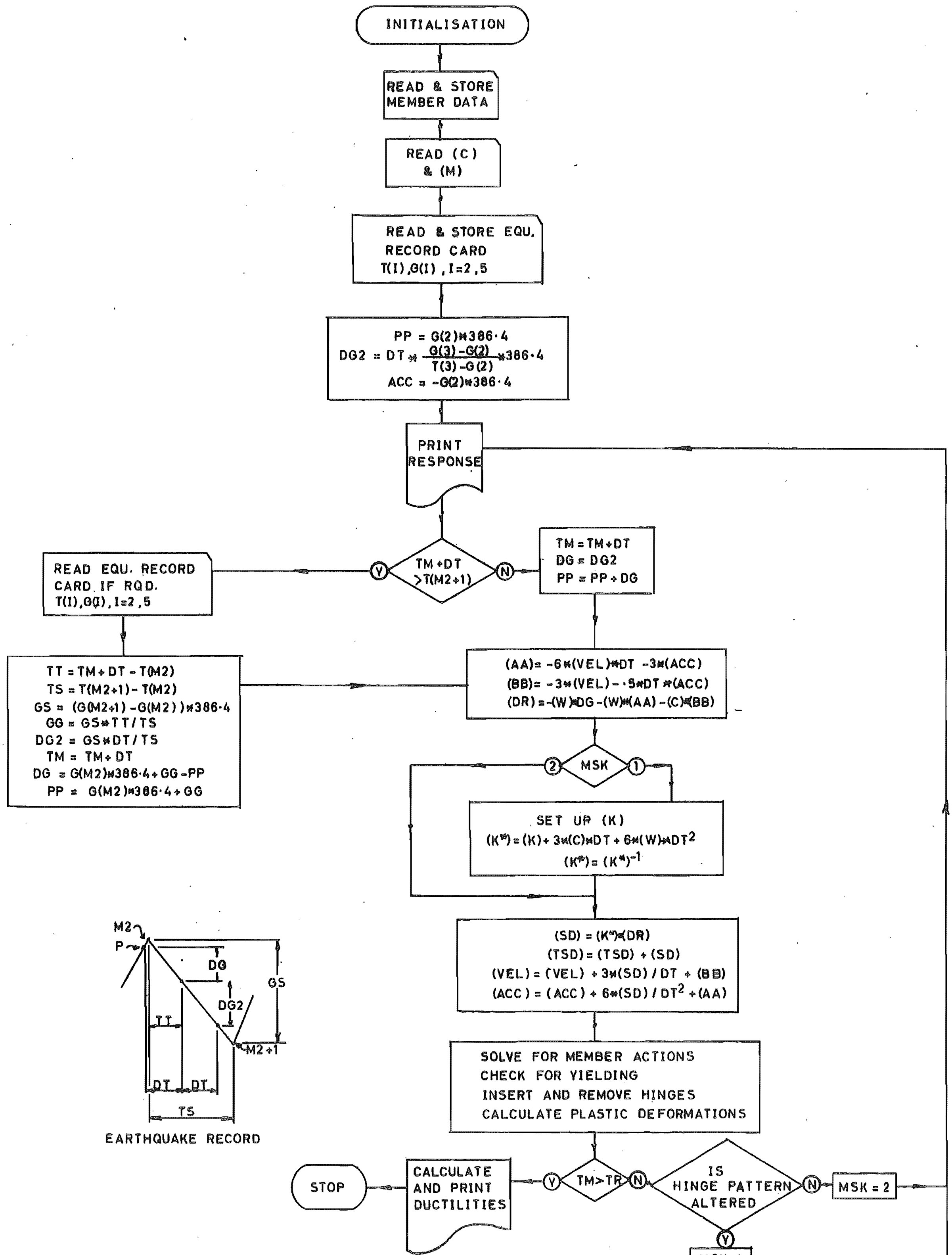


FIG. 4·1

PROGRAM "DYNEPRES"
BLOCK DIAGRAM
FOR LIST 10



```
C     PROGRAM DOUBLE PRECISION DYNEPRES.
C     LANGUAGE - FORTRAN IV FOR IBM 360/44
C
C     THIS PROGRAM CALCULATES THE ELASTO/PLASTIC RESPONSE OF A REGULAR
C     MULTI-STOREY FRAME TO AN EARTHQUAKE RECORD.
C
      DIMENSION T(5),G(5),TSD(42),SD(43),VEL(14),ACC(14),S(42,42),
     1C(14,14),AA(14),BB(14),W(14),DR(42),IS(56),JS(56),KS(56),LS(56),
     2P1(56),Q1(56),YM1(56),YM2(56),IH1(56),IH2(56),PD1(56),PD2(56),
     3PDM1(56),PDM2(56),R1(56),R2(56),IP1(56),IP2(56)
      DOUBLE PRECISION TR,E,XMAX,P,R,Q,T2,TMAX,U1,U2,
     1DT,TM,PP,DG2,DG,TT,TS,GS,GG,T,G,TSD,SD,VEL,ACC,
     2S,C,AA,BB,W,DR,P1,Q1,YM1,YM2,PD1,PD2,PDM1,PDM2,R1,R2,GRAV
      INP=5
      LNP=6
      IPCH=7
C     N=NO. OF FRAME DEFORMATIONS,NS=NO. OF STOREYS,NB=NO.OF BAYS,
C     M4=NO OF MEMBERS, M3=NO OF MEMBERS WHICH MAY YIELD,
C     KP=NO OF STEPS BEFORE PRINTING, DT=STEP INTERVAL,
C     TR=LENGTH OF EQU. RECORD, E=YOUNGS MODULUS
C     GRAV=GRAVITY 386.4, USE OF DIFFERENT VALUE SCALES EQU. RECORD.
  731 READ(INP,30) N,NS,NB,M4,M3,KP,DT,TR,E,GRAV
   30 FORMAT(6I3,4E10.4)
C     I1-I7,J1-J7 CONTROL PRINT AND PUNCH OUTPUT
      READ(INP,34) I1,I2,I3,I4,I5,I6,I7
      READ(INP,34) J1,J2,J3,J4,J5,J6,J7
   34 FORMAT(8I3)
      SD(N&1)=0.
      KC=KP
      NB2=NB&2
      NB1=NB&1
      ISQ=1
      DO171M=1,M3
      PD1(M)=0.
  171 PD2(M)=0.
      WRITE(LNP,330) N,NS,NB,M4,M3,KP,DT,TR,E,GRAV
  330 FORMAT(1X,6I4,4E13.4///)
      WRITE(LNP,346) I1,I2,I3,I4,I5,I6,I7
      WRITE(LNP,346) J1,J2,J3,J4,J5,J6,J7
  346 FORMAT(1X,8I4)
C     READ AND STORE MEMBER DATA
C     IS,JS,KS,LS=DEFORMATIONS ASSOCIATED WITH MEMBER
C     P1=MOMENT OF INERTIA,Q1=LENGTH,YM1,YM2=YIELD MOMENTS
C     REDUCED BY 1.E3,IH1&IH2=1 OR 0 FOR HINGE AT ENDS OR NOT.
C     R1,R2 INITIAL END MOMENTS.
      DO28M=1,M4
      READ(INP,31) IS(M),JS(M),KS(M),LS(M),P1(M),Q1(M),YM1(M),YM2(M),
     1IH1(M),IH2(M),R1(M),R2(M)
   31 FORMAT(4I3,4E6.0,2I2,4E10.0)
      WRITE(LNP,331) IS(M),JS(M),KS(M),LS(M),P1(M),Q1(M),YM1(M),YM2(M),
     1IH1(M),IH2(M),R1(M),R2(M)
  331 FORMAT(4I4,4E13.4,2I3,3E13.4)
      YM1(M)=YM1(M)*1.E3
      YM2(M)=YM2(M)*1.E3
   28 P1(M)=P1(M)*E/Q1(M)
      WRITE(LNP,341)
C     READ DAMPING MATRIX
      DO29I=1,NS
      READ(INP,32) (C(I,J),J=1,NS)
      WRITE(LNP,332) (C(I,J),J=1,NS)
```

LIST 10.


```
   29 WRITE(LNP,340)
   32 FORMAT(5E14.8)
  332 FORMAT(1X,5E18.8)
  340 FORMAT(2X)
      WRITE(LNP,341)
  341 FORMAT(//)
C     READ MASS MATRIX
      READ(INP,33) (W(I),I=1,NS)
   33 FORMAT(8E10.4)
      WRITE(LNP,333) (W(I),I=1,NS)
  333 FORMAT(1X,8E13.4/)
      WRITE(LNP,341)
      DO40I=1,NS
   40 W(I)=W(I)/386.4
      DO41I=1,M3
      PDM1(I)=0.
      PDM2(I)=0.
      IP1(I)=IH1(I)
   41 IP2(I)=IH2(I)
      DO42I=1,N
   42 TSD(I)=0.
      DO43I=1,NS
   43 VEL(I)=0.
      TM=0.
      XMAX=0.
      MSK=1
      ICC=1
      READ(INP,35) ISC,(T(I),G(I),I=2,5)
   35 FORMAT(I3,4(F8.4,F9.6))
      IF(ICC-ISC)44,45,44
   45 M2=2
      PP=G(M2)*GRAV
      DG2=(G(3)-G(2))/(T(3)-T(2))*DT*GRAV
      DO46I=1,NS
   46 ACC(I)=-G(2)*GRAV
   47 KC=KC&1
      IF(KC-KP)356,357,357
  357 KC=0
      WRITE(LNP,340)
C     PRINT LATERAL DISPLACEMENTS.
      WRITE(LNP,334) TM,(TSD(I),I=NB2,N,NB2)
  334 FORMAT(1X,8E13.4)
      IF(I7-1)555,142,143
  143 WRITE(IPCH,344) TM,TSD(I1),TSD(I2),TSD(I3),TSD(I4),TSD(I5),TSD(I6)
     1,ISQ
  344 FORMAT(7E10.3,5X,I5)
      WRITE(IPCH,342) R1(J1),R1(J2),R1(J3),R2(J4),R1(J5),R2(J6),R2(J7),
     1ISQ
  342 FORMAT(7E10.3,I5)
  142 WRITE(LNP,335) R1(J1),R1(J2),R1(J3),R2(J4),R1(J5),R2(J6),R2(J7),
     1ISQ
      WRITE(LNP,334)PD1(J1),PD1(J2),PD1(J3),PD2(J4),PD1(J5),PD2(J6)
     1,PD2(J7)
  335 FORMAT(1X,7E13.4,10X,I4)
  555 ISQ=ISQ&1
  356 IF(TM&DT-T(M2&1))48,48,49
   48 TM=TM&DT
      DG=DG2
      PP=PP&DG
      GO TO 50
```

```
   49 IF(M2-4)80,81,81
   80 M2=M2&1
      GO TO 83
   81 M2=1
      G(1)=G(5)
      T(1)=T(5)
      ICC=ICC&1
C     READ BERG EQU. RECORD CARD.
      READ(INP,35) ISC,(T(I),G(I),I=2,5)
      IF(ICC-ISC)44,83,44
   44 WRITE(LNP,337) ICC
  337 FORMAT(1X,19HCARDS OUT OF ORDER.I5)
      STOP
   83 TT=TM&DT-T(M2)
      TS=T(M2&1)-T(M2)
      IF(TS)49,49,84
   84 GS=(G(M2&1)-G(M2))*GRAV
      GG=GS*TT/TS
      DG2=GS*DT/TS
      TM=TM&DT
      DG=G(M2)*GRAV&GG-PP
      PP=G(M2)*GRAV&GG
   50 DO85I=1,NS
      AA(I)=-6.0*VEL(I)/DT-3.0*ACC(I)
   85 BB(I)=-3.0*VEL(I)-0.5*DT*ACC(I)
      DO86I=1,NS
      DO82 K=1,NB1
      L=(I-1)*NB2&K
   82 DR(L)=0.
      GS=0.
      DO87J=1,NS
   87 GS=GS&C(I,J)*BB(J)
      K=I*NB2
   86 DR(K)=-W(I)*DG-W(I)*AA(I)-GS
      GO TO (88,89),MSK
C     SET UP FRAME STIFFNESS MATRIX
   88 WRITE(LNP,340)
      WRITE(LNP,333) TM
      WRITE(LNP,106) (IH1(I),IH2(I),I=1,M3)
  106 FORMAT(1X,36I3)
      DO140I=1,N
      DO140J=1,N
  140 S(I,J)=0.
      DO27M=1,M4
      I=IS(M)
      J=JS(M)
      K=KS(M)
      L=LS(M)
      P=P1(M)
      IF(IH1(M)-1)3,4,4
    3 IF(IH2(M)-1)5,8,8
    4 IF(IH2(M)-1)6,27,27
    5 R=4.0
      Q=6.0/Q1(M)
      GO TO 7
    8 K=N&1
      GO TO 26
    6 I=N&1
   26 R=3.0
      Q=3.0/Q1(M)
```



```
    7 IF(I-N)9,9,10
    9 S(I,I)=S(I,I)&R*P
      IF(J-N)11,11,12
   11 S(I,J)=S(I,J)&Q*P
      S(J,I)=S(I,J)
   12 IF(K-N)13,13,14
   13 S(I,K)=S(I,K)&2.0*P
      S(K,I)=S(I,K)
   14 IF(L-N)15,15,16
   15 S(I,L)=S(I,L)-Q*P
      S(L,I)=S(I,L)
   10 IF(L-N)17,17,16
   17 S(L,L)=S(L,L)&Q*Q*P*.33333333
      IF(J-N)18,18,19
   18 S(J,L)=S(J,L)-Q*Q*P*.33333333
      S(L,J)=S(J,L)
   19 IF(K-N)20,20,16
   20 S(K,L)=S(K,L)-Q*P
      S(L,K)=S(K,L)
   16 IF(J-N)21,21,22
   21 S(J,J)=S(J,J)&Q*Q*P*.33333333
      IF(K-N)23,23,27
   23 S(J,K)=S(J,K)&Q*P
      S(K,J)=S(J,K)
   22 IF(K-N)24,24,27
   24 S(K,K)=S(K,K)&R*P
   27 CONTINUE
      DO90I=1,NS
      K=I*NB2
      S(K,K)=S(K,K)&6.0*W(I)/(DT*DT)
      DO90J=1,NS
      L=J*NB2
   90 S(K,L)=S(K,L)&3.0*C(I,J)/DT
C     INVERTS (S) IN SITU. ORDER NXN.
      DO101I=1,N
      T2=S(I,I)
      IF(T2)350,351,350
  351 WRITE(LNP,339)
  339 FORMAT(1X,15HSINGULAR MATRIX////)
      STOP
  350 S(I,I)=1.0
      DO102J=1,N
  102 S(I,J)=S(I,J)/T2
      DO101L=1,N
      IF(L-I)103,101,103
  103 T2=S(L,I)
      S(L,I)=0.
      DO104J=1,N
  104 S(L,J)=S(L,J)-T2*S(I,J)
  101 CONTINUE
   89 DO91I=1,N
      GS=0.
      DO92J=1,N
   92 GS=GS&S(I,J)*DR(J)
      SD(I)=GS
   91 TSD(I)=TSD(I)&SD(I)
      DO93I=1,NS
      J=I*(NB&2)
      VEL(I)=VEL(I)&3.0*SD(J)/DT&BB(I)
   93 ACC(I)=ACC(I)&6.0*SD(J)/(DT*DT)&AA(I)
```

```
      IF(DABS(TSD(N)).LE.DABS(XMAX)) GO TO 112
  113 XMAX=TSD(N)
      TMAX=TM
C     SOLVE FOR MEMBER END ACTIONS. CHECK FOR YIELDING.
C     ADD OR REMOVE HINGES.
  112 DO78M=1,M3
      I=IS(M)
      J=JS(M)
      K=KS(M)
      L=LS(M)
      P=P1(M)
      IF(IH1(M)-1)51,52,52
   51 IF(IH2(M)-1)53,54,54
   52 IF(IH2(M)-1)55,56,56
   54 R1(M)=R1(M)&(3.0*SD(I)&3.0/Q1(M)*(SD(J)-SD(L)))*P
      Q=SD(K)&0.5*SD(I)-1.5*(SD(L)-SD(J))/Q1(M)
      MM=1
   58 IF(R2(M)*Q)60,62,62
   60 IH2(M)=0
      GO TO 66
   62 PD2(M)=PD2(M)&Q
      IF(DABS(PD2(M))-DABS(PDM2(M)))66,66,65
   65 PDM2(M)=PD2(M)
   66 GO TO (166,67),MM
  166 IF(DABS(R1(M))-YM1(M))63,63,76
   76 IH1(M)=1
      R1(M)=.9999999*YM1(M)*R1(M)/DABS(R1(M))
   63 GO TO 78
   55 R2(M)=R2(M)&(3.0*SD(K)&3.0/Q1(M)*(SD(J)-SD(L)))*P
      Q=SD(I)&0.5*SD(K)-1.5*(SD(L)-SD(J))/Q1(M)
      MM=1
   68 IF(R1(M)*Q)72,74,74
   72 IH1(M)=0
      GO TO 79
   74 PD1(M)=PD1(M)&Q
      IF(DABS(PD1(M))-DABS(PDM1(M)))79,79,75
   75 PDM1(M)=PD1(M)
   79 GO TO (179,78),MM
  179 IF(DABS(R2(M))-YM2(M))78,78,170
  170 IH2(M)=1
      R2(M)=.9999999*YM2(M)*R2(M)/DABS(R2(M))
      GO TO 78
   56 Q=SD(K)-(SD(L)-SD(J))/Q1(M)
      MM=2
      GO TO 58
   67 Q=SD(I)-(SD(L)-SD(J))/Q1(M)
      GO TO 68
   53 R1(M)=R1(M)&(4.0*SD(I)&2.0*SD(K)&6.0/Q1(M)*(SD(J)-SD(L)))*P
      R2(M)=R2(M)&(2.0*SD(I)&4.0*SD(K)&6.0/Q1(M)*(SD(J)-SD(L)))*P
      IF(DABS(R1(M))-YM1(M))179,179,77
   77 IH1(M)=1
      R1(M)=.9999999*YM1(M)*R1(M)/DABS(R1(M))
      GO TO 179
   78 CONTINUE
      IF(TM-TR&.5*DT)94,95,95
   94 MSK=2
      DO100I=1,M3
      IF(IH1(I)-IP1(I))198,99,198
   99 IF(IH2(I)-IP2(I))198,199,198
  198 MSK=1
```



```
  199 IP1(I)=IH1(I)
  100 IP2(I)=IH2(I)
      GO TO 47
   95 WRITE(LNP,345)
C     PRINT MAX TOP STOREY DEFLECTION AND TIME IT OCCURRED
      WRITE(LNP,333) XMAX,TMAX
      WRITE(LNP,341)
      DO96I=1,M3
      IF(IH2(I)-1)500,501,501
  500 U2=1.0&DABS(PDM2(I))*P1(I)*6.0/YM2(I)
      U1=1.0&DABS(PDM1(I))*P1(I)*6.0/YM1(I)
C     PRINT MEMBER DUCTILITIES.
      WRITE(LNP,336) I,U1,U2
      GO TO 96
  501 U1=1.0&DABS(PDM1(I))*P1(I)*3.0/YM1(I)
      WRITE(LNP,336) I,U1
   96 CONTINUE
  336 FORMAT(1X,I3,2F10.2)
      READ(INP,730) NFIN
  730 FORMAT(I3)
      GO TO (733,732),NFIN
  733 WRITE(LNP,345)
  345 FORMAT(1H1)
      GO TO 731
  732 WRITE(LNP,338)
  338 FORMAT(//1X,19HPROCESSING COMPLETE////)
      STOP
      END
```

CHAPTER FIVE

ELASTO-PLASTIC ANALYSIS OF TWO MULTI-STOREY FRAMES

BUILDING A

5.1 INTRODUCTION

This section considers the dynamic analysis of the behaviour of a 14 storey steel frame building when excited by a large digital earthquake record. The 1940 El Centro earthquake, N-S component, was used intensified by a factor of 50% to bring it up to the magnitude of the largest earthquake considered possible(23), having a maximum acceleration of 0.50g. The building was designed to be constructed in Wellington, New Zealand, and is symmetrical in plan having five bays in the longitudinal direction and three bays in the transverse with a fundamental period of approximately 2.80 secs in both directions. The structure is very flexible and the object of the analysis was to determine the magnitude and distribution of the plastic deformations in the members of the frame and to see if a major earthquake would cause a significant amount of plastic drift.

The building has 14 main floors of area 11,000 sq.ft. with two set-back floors on the top and two basement floors which are part of massive reinforced concrete vaults. An interior transverse frame was analysed and for this purpose was simplified to a regular fourteen storey, three bay frame rigidly fixed at ground level, the mass of the set-back floors being lumped at the fourteenth floor, and the extra bays forming a podium at the lower two floors being neglected when lateral resistance was considered because they were of much lighter sections. The properties of the members and the floor masses are shown in Figure 5.1, the moments of inertia being calculated from the uncased steel sections because it was proposed to provide the fire protection with sprayed asbestos.

The New Zealand code NZSS 1900, Chapter 8[5], prescribes the lateral static design loads in terms of a seismic coefficient which is a function of the fundamental period of the structure. The code requirements have been derived from the elastic response spectrum curve for the spectral acceleration of a single mass resonator with 10% damping excited by a major earthquake. A reduction factor of 4.0 has been used for short period structures because it was intended that plastic action should be called on to resist the major tremors, but a factor of approximately 2.0 has been used for long period structures. Thus although a flexible steel frame is capable of providing a high degree of ductility before total collapse occurs this property has not been fully used because of the desirability of limiting both the elastic inter-storey deflection under small earthquakes and the plastic drift which may occur under large earthquakes.

The elastic and elasto-plastic responses to the intensified El Centro record are described and a comparative static analysis is made with the code forces and those predicted by a normal mode-response spectrum analysis.

5.2 ELASTIC ANALYSIS

A normal mode analysis was carried out to enable the seismic forces to be estimated from a response spectrum. The member stiffnesses were estimated from the steel sections, taking Young's Modulus as 30×10^6 lb/in^2, the lateral flexibility matrix being calculated by program "KLAT" which considered bending deformations only as the effects of shear deformation, axial deformation and joint size were considered to be sufficiently small to be neglected. The seismic forces were predicted by taking the root mean square of the modal responses found using the response curve assuming 5% critical damping(24), with an intensification factor of 1.5. The member actions caused by these predicted forces were found by the program "T.D.E." and are shown in Figure 5.2, together with the lateral forces. The normal mode properties and the predicted response are shown in Table 5.1.

The elastic response to the intensified El Centro record was found by integrating the modal equations of motion and summing the responses at each increment using the program "SUMMOD". The variation of the displacements of the floors with time is shown in Figure 5.3, and the maximum floor displacements are plotted in Figure 5.2.

A static analysis was also carried out for comparative purposes to determine the member actions under the lateral loading required by N.Z. code, NZSS 1900 Chapter 8[5]. Lateral loads were found using a coefficient of 0.08 on the base shear and distributed as required by the code, i.e. in approximately triangular fashion. The lateral loads, deflections and member actions are compared with those predicted from the dynamic elastic response in Figure 5.2. A factor of 2.6 is needed to reduce the predicted elastic response to the code figures, comparing base shear values.

5.3 ELASTO-PLASTIC ANALYSIS

The elasto-plastic response of the 14 storey 3 bay frame to the intensified El Centro record was found with the computer program "DYNEPRES". The ultimate moments of the members were calculated using a yield stress of 36,000 lb/in^2 and a shape factor of 1.15 for the beams and 1.00 for the columns, to allow for the effects of axial load approximately. The values assumed for the exterior beams and columns are given in Figure 5.2. The damping was assumed to be 5% of critical in the first two modes using the simplified assembly of the damping matrix (C) as detailed earlier.

The variation of displacement of the floors against time is shown in Figure 5.4. The plastic deformation reduces the maximum top storey deflection by approximately 50% and also caused the periodicity of motion to become a little longer. The displacements are similar in magnitude up to the peak at 3.3 secs, but after that the plastic action dissipates energy and prevents the build-up of elastic strain and kinetic energy which would lead to an increase of response. Figure 5.4 also shows that a significant amount of permanent deformation has occurred and that the top floor would be left with a permanent set of several inches beyond the base.

Thus even though the required ductilities of the individual members are quite modest the elastic inter-storey deflection under small earthquakes and the plastic drift which occurs under large earthquakes are significant and with the present state of knowledge of the actual behaviour in an earthquake it would not be desirable to reduce the required design seismic coefficient for long period structures.

Figure 5.2 shows that the beam yield moments are set much lower than the column yield moments relative to the predicted elastic response, and so it would be expected that more yielding would take place in the beams. The only yielding in the columns occurs at the base of the bottom column, most of the building's energy absorption being provided by the beams. The required ductilities of the exterior beams are compared with the moment ratios in Figure 5.2. The moment ratio is defined as the ratio of the maximum bending moment assuming elastic behaviour to the yield moment of the same section.

It can be seen that a high ductility is required near the bottom of the building where a high moment ratio was obtained and this peak in the moment-ratio plot is accentuated in the ductility requirements. High ductilities are also required around the 9th floor; this behaviour was also noted by Clough, Benuska and Wilson (16) and is presumably caused by the effects of modes of vibration similar to the second and third elastic modes.

The variation of bending moment with time and the growth of plastic deformation as yielding occurs is shown in Figure 5.5. It can be seen that the plastic deformation changes over a comparatively short time corresponding to the rotation of a plastic hinge. The effect of the plastic action is to cut off the peak of the moment curve giving a distinctive flat crest to the plot. It should be remembered that the member is plastic only on the flat crest, either side the member is elastic and the plastic action does not appear to alter the shape of the moment curve either side of the crest.

TABLE 5.1

NORMAL MODE PROPERTIES

Mode 1

Frequency = 0.362 c.p.s. Period = 2.765 secs.

Amplification Factor = 0.23

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	99.39	1016
.965	95.93	1534
.912	90.65	2023
.844	83.89	2476
.768	76.29	2888
.686	68.20	3256
.599	59.56	3577
.515	51.18	3853
.435	43.22	4087
.355	35.28	4277
.281	27.93	4428
.212	21.09	4542
.127	12.65	4610
.053	5.26	4638

Mode 2

Frequency = 0.998 c.p.s. Period = 1.001 secs.

Amplification Factor = 0.64

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	-4.68	-365
.749	-3.50	-509
.395	-1.85	-585
-.002	.01	-584
-.371	1.73	-513
-.666	3.11	-385
-.878	4.11	-216
-.989	4.62	-258
-1.003	4.69	167
-.938	4.39	347
-.818	3.83	505
-.657	3.07	631
-.413	1.93	711
-.177	.83	745

TABLE 5.1 continued

Mode 3

Frequency = 1.75 c.p.s. Period = 0.572 secs.

Amplification Factor = 1.13

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	.734	175.4
.312	.229	204.3
-.497	-.365	158.3
-1.117	-.820	54.9
-1.335	-.980	-68.7
-1.114	-.817	-171.7
-.574	-.421	-224.9
.059	.043	-219.4
.614	.450	-162.6
1.007	.739	-69.4
1.169	.858	38.8
1.096	.804	140.2
.772	.567	211.7
.353	.259	244.3

PREDICTED ELASTIC RESPONSE

Displacement inches	Shears Kips	Force Kips
15.95	265	264.6
15.34	366	101.7
14.44	430	63.1
13.36	472	42.6
12.18	515	42.4
10.95	561	46.2
9.65	602	41.7
8.39	636	33.6
7.19	666	30.2
5.96	699	33.3
4.80	739	39.7
3.67	781	42.3
2.23	814	32.1
.93	829	15.2

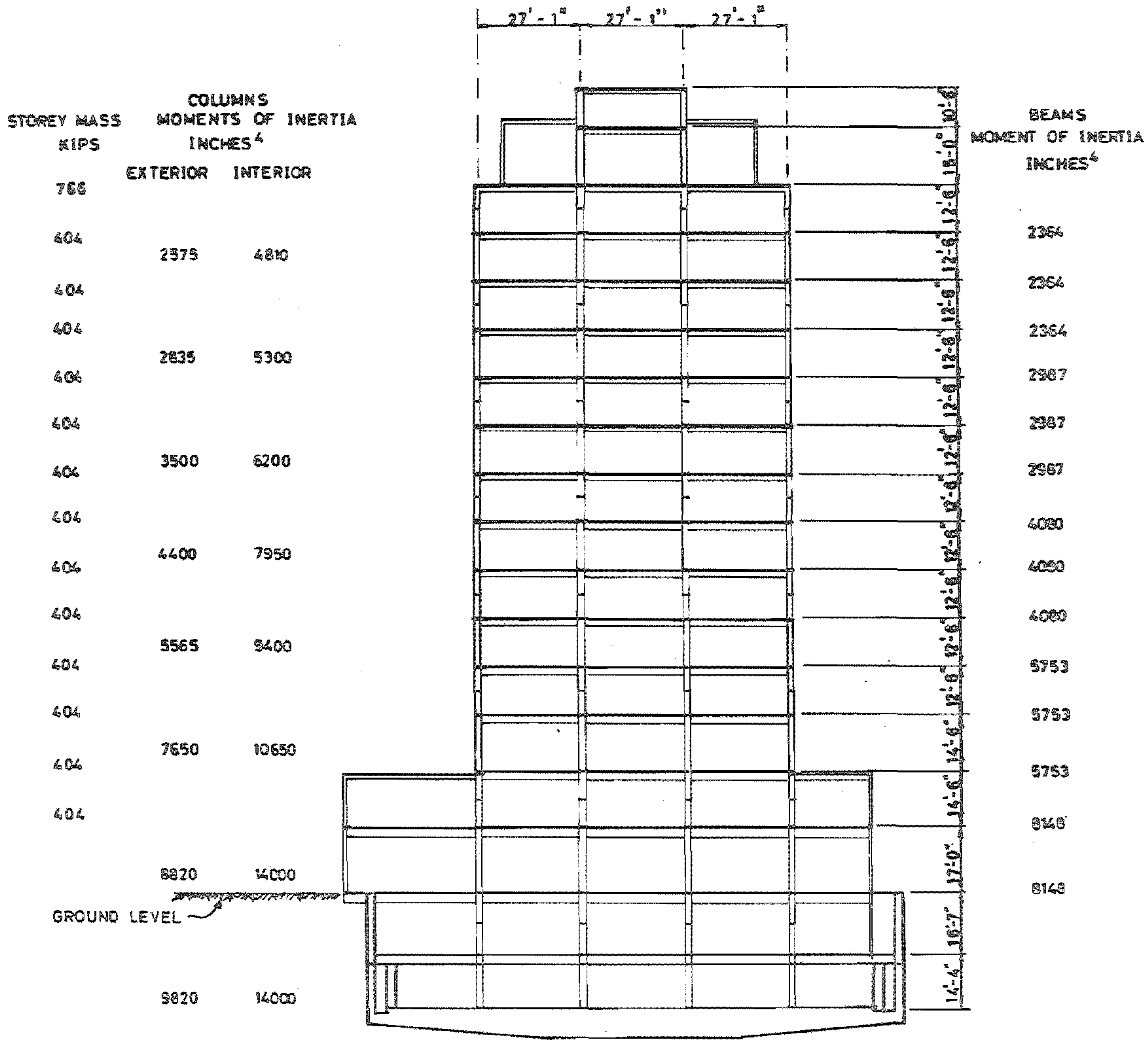

27'-1"
27'-1"
27'-1"
STOREY MASS
KIPS
COLUMNS
MOMENTS OF INERTIA
INCHES4
EXTERIOR
INTERIOR
766
404
2575
4810
404
404
2835
5300
404
404
3500
6200
404
404
4400
7950
404
404
5565
9400
404
404
7650
10650
404
404
8820
14000
GROUND LEVEL
9820
14000
10'-6"
15'-0"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
12'-6"
14'-6"
14'-6"
17'-0"
16'-7"
14'-4"
BEAMS
MOMENT OF INERTIA
INCHES4
2364
2364
2364
2987
2987
2987
4080
4080
4080
5753
5753
5753
8148
8148

FIGURE 5-1

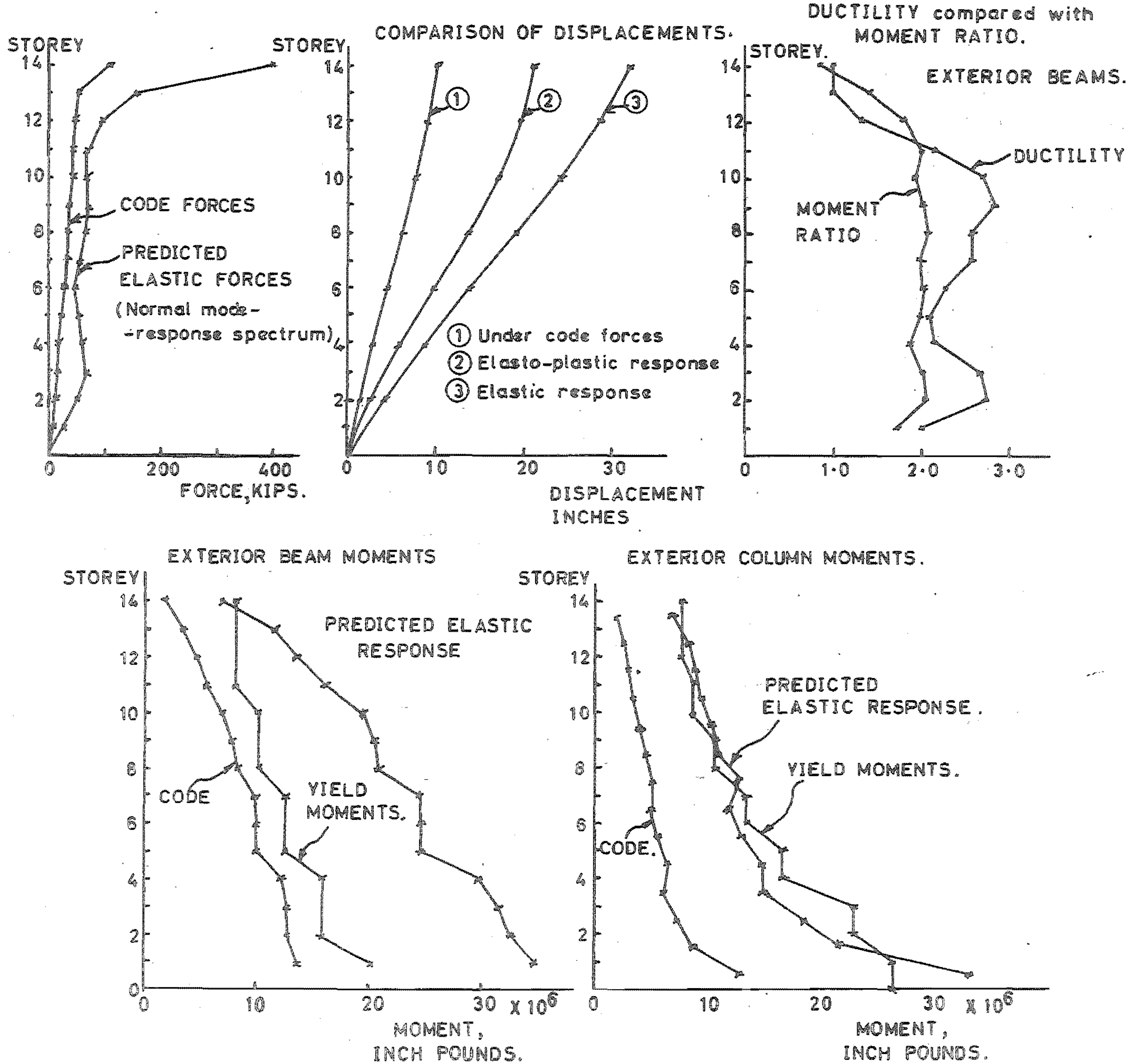
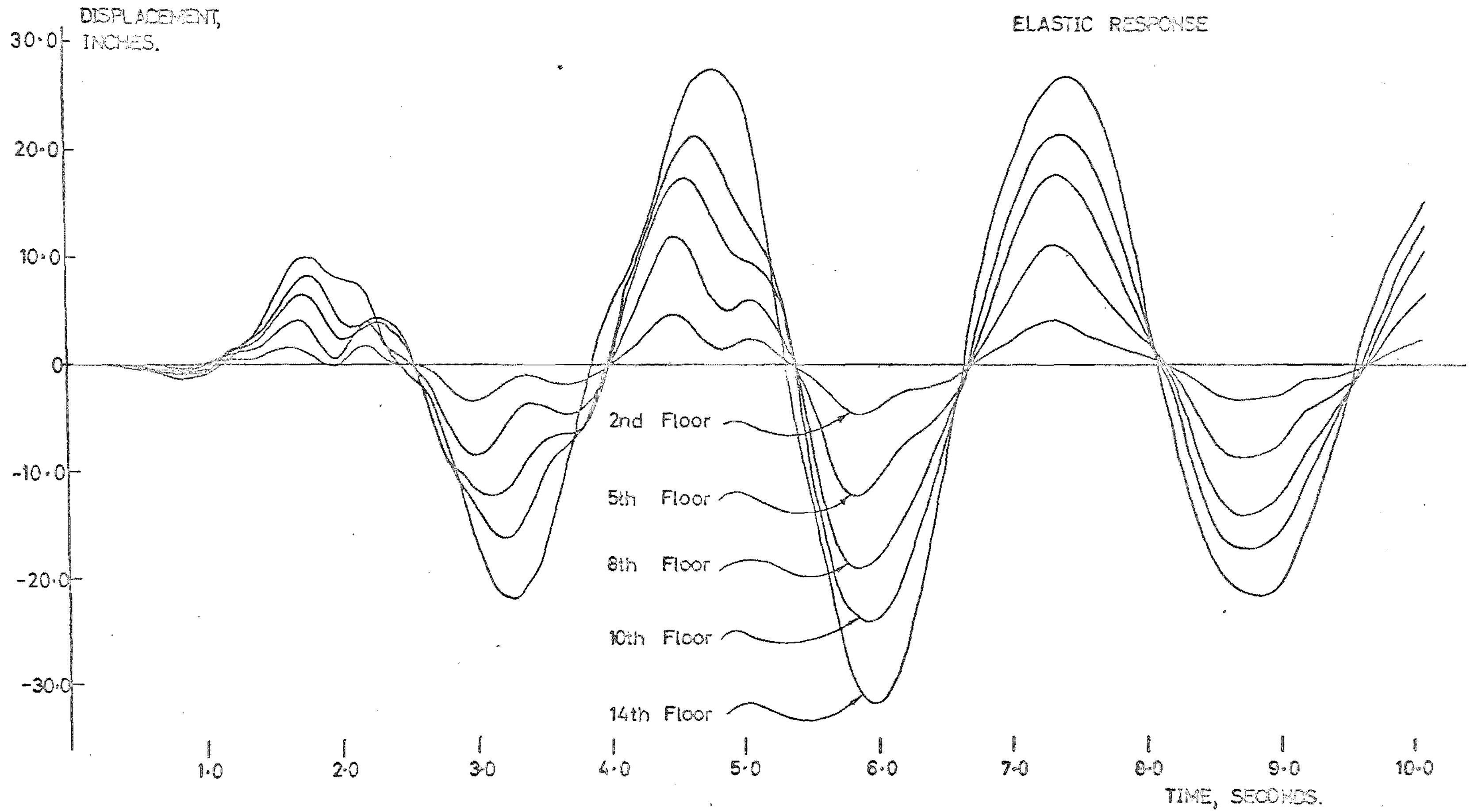

STOREY
CODE FORCES
PREDICTED ELASTIC FORCES (Normal mode--response spectrum)
FORCE, KIPS.
COMPARISON OF DISPLACEMENTS.
① Under code forces
② Elasto-plastic response
③ Elastic response
DISPLACEMENT INCHES
DUCTILITY compared with MOMENT RATIO.
EXTERIOR BEAMS.
DUCTILITY
MOMENT RATIO
EXTERIOR BEAM MOMENTS
PREDICTED ELASTIC RESPONSE
CODE
YIELD MOMENTS.
MOMENT, INCH POUNDS.
EXTERIOR COLUMN MOMENTS.
PREDICTED ELASTIC RESPONSE.
YIELD MOMENTS.
CODE.

FIGURE 5.2



DISPLACEMENT, INCHES.
ELASTIC RESPONSE
30·0
20·0
10·0
0
-10·0
-20·0
-30·0
2nd Floor
5th Floor
8th Floor
10th Floor
14th Floor
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 5·3

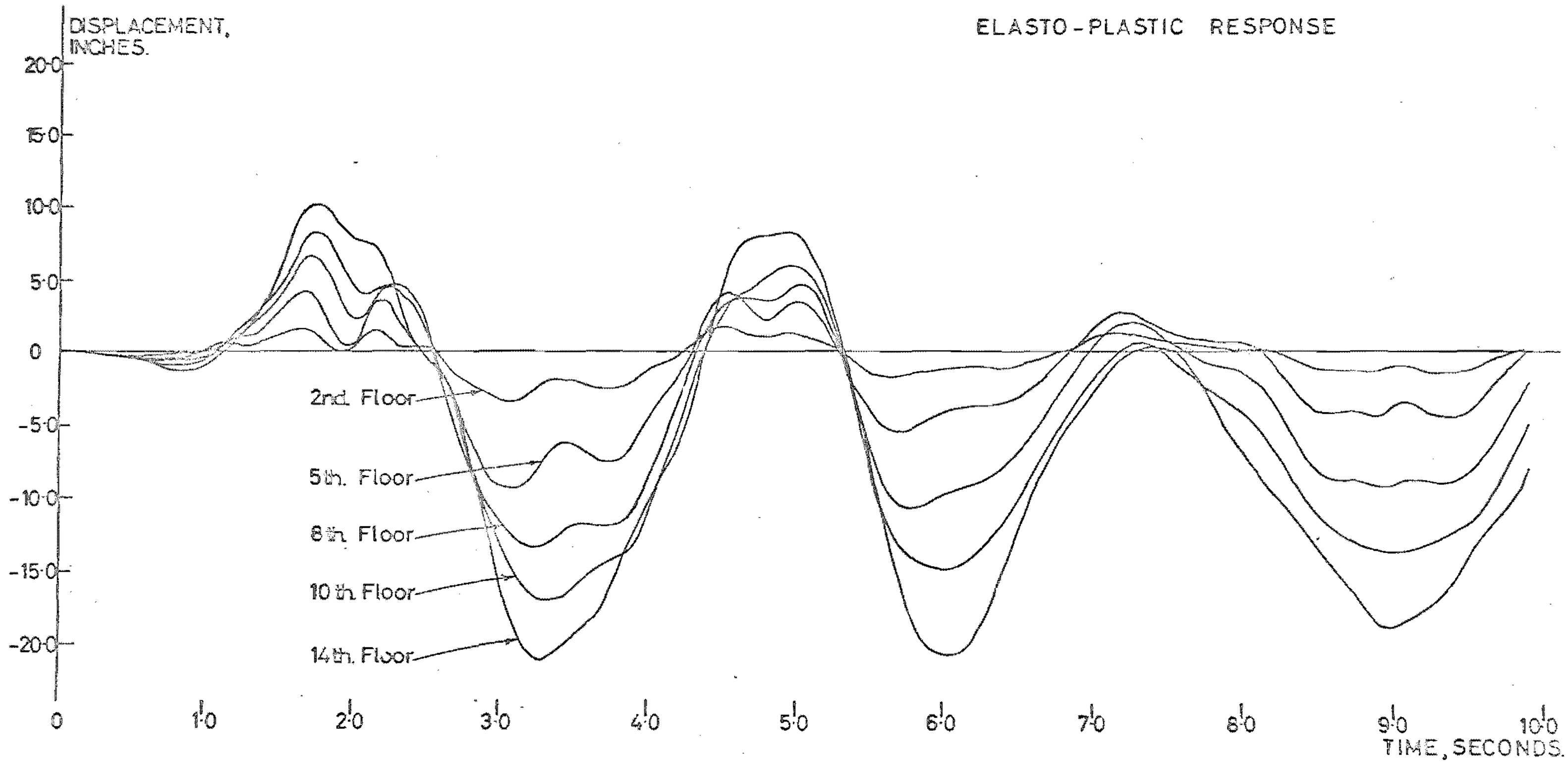

ELASTO - PLASTIC RESPONSE
DISPLACEMENT, INCHES.
20·0
15·0
10·0
5·0
0
-5·0
-10·0
-15·0
-20·0
2nd. Floor
5th. Floor
8th. Floor
10 th. Floor
14th. Floor
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 5.4

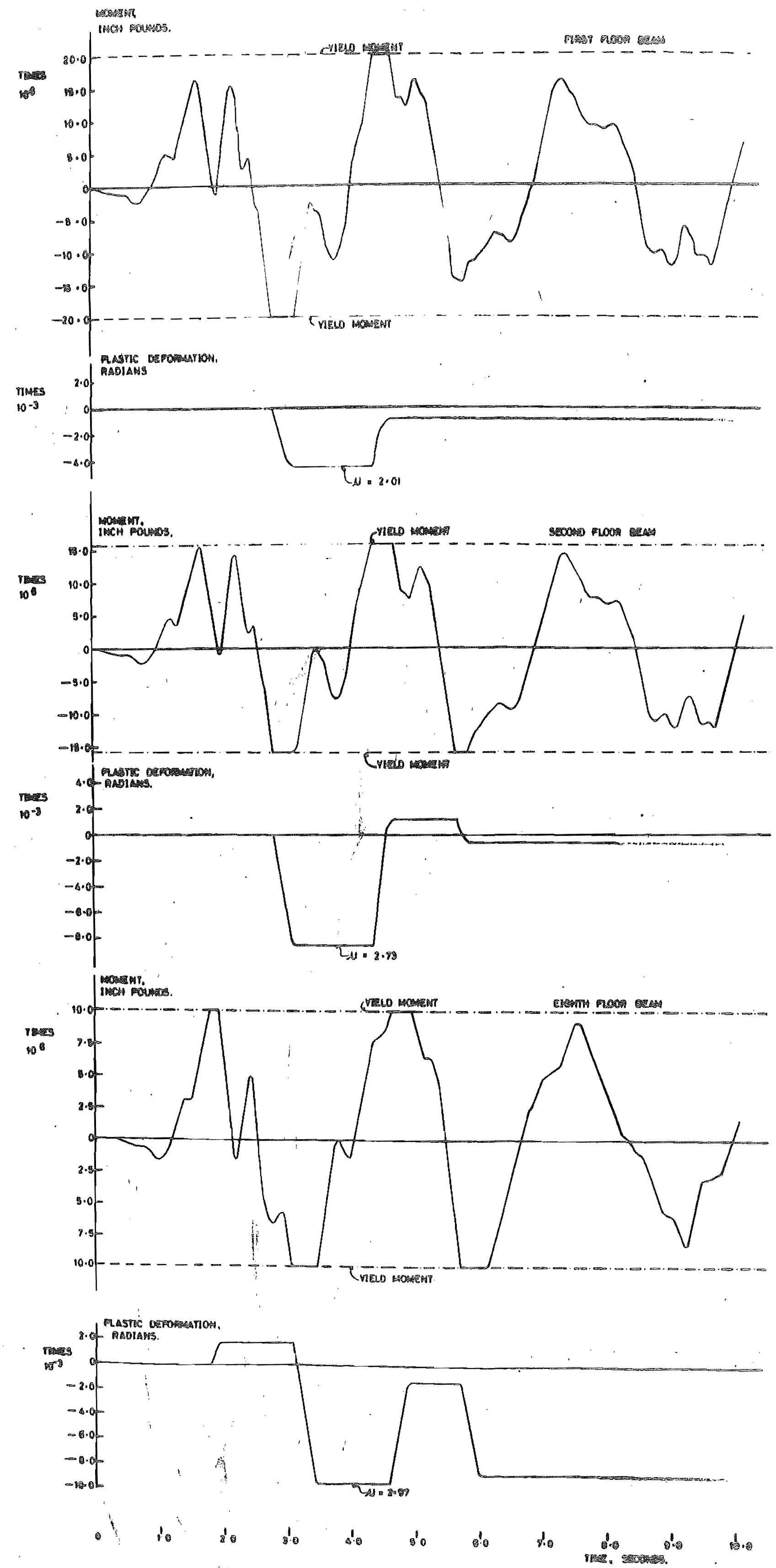

MOMENT, INCH POUNDS.
YIELD MOMENT
FIRST FLOOR BEAM
TIMES 10^6
YIELD MOMENT
PLASTIC DEFORMATION, RADIANS
TIMES 10^{-3}
μ = 2·01
MOMENT, INCH POUNDS.
YIELD MOMENT
SECOND FLOOR BEAM
TIMES 10^6
YIELD MOMENT
PLASTIC DEFORMATION, RADIANS.
TIMES 10^{-3}
μ = 2·79
MOMENT, INCH POUNDS.
YIELD MOMENT
EIGHTH FLOOR BEAM
TIMES 10^6
YIELD MOMENT
PLASTIC DEFORMATION, RADIANS.
TIMES 10^{-3}
TIME, SECONDS.

FIGURE 5·5

BUILDING B

5.4 THE STRUCTURE

The reinforced concrete structure is six storeys high and has 11 bays of 12'0" width in the longitudinal direction and one bay of 33'0" width in the transverse direction. Five of the central transverse frames have an additional bay added, with members of slender proportions. This was neglected when the overall lateral resistance of the building was considered. The seismic resistance of the building is provided entirely by the reinforced concrete frames as there are no shear walls present. The proportions of and member properties for a typical transverse frame are shown in Figure 5.6. An outline of the elastic and elasto-plastic behaviour of this frame under the 1940 El Centro earthquake N-S component is given in the next two sections.

5.5 ELASTIC ANALYSIS

A normal mode analysis was carried out enabling the seismic forces to be predicted using Skinner's Response Spectra[24]. The member stiffnesses were estimated using the gross concrete areas, taking Young's Modulus as 5×10^6 lb/in^2. The lateral flexibility matrix was calculated using the computer program "KLAT" which considered the effects of shear deformation, axial deformation and joint size. The seismic forces were determined by taking the root mean square of the modal responses found using the response spectrum curve for 5% critical damping. The member actions caused by these predicted forces were found by another computer program and are shown in Figure 5.7, together with the lateral forces. The normal mode properties and predicted response are shown in Table 5.2.

The elastic response to the El Centro 1940 N-S earthquake record was found by integrating the modal equations of motion and summing the response at each increment using the computer program "SUMMOD". The displacements predicted from this analysis are shown in Figure 5.10. The variation of the lateral displacement of the various storeys is plotted against time, assuming 10% critical damping, in Figure 5.8.

A static analysis was also carried out to determine the member actions under the lateral loading required by the N.Z. code 1900 Chapter 8. Lateral loads were found using a coefficient of 0.15 on the base shear and distributed in proportion to the height above base level. The coefficient of 0.15 corresponds to a public building with a fundamental period of vibration of 0.5 seconds constructed in seismic zone A of New Zealand. The member actions and lateral loads are shown in Figure 5.7. A factor of 4.2 is needed to reduce the predicted elastic response to the code figures, comparing base shear values.

5.6 ELASTO-PLASTIC ANALYSIS

The elasto-plastic response of the frame shown in Figure 5.6

to the 1940 El Centro earthquake, N-S component, was determined using a computer program run on a C.D.C.3600 machine.

The ultimate moments of the beam and column sections were calculated using Whitney[25] theory and the philosophy of the ACI code 318-63[26] as stated in section 1503.

The following assumptions were made:

1. An ultimate concrete stress, fc', of 4000 lb/sq.in. and a steel yield stress, fy, of 36,000 lb/sq.in. was assumed.

2. A rectangular stress block with an average ultimate stress of 0.85 fc' was assumed.

In addition for the column sections:

3. The strain in the reinforcing bars was assumed to be equal to the strain in the concrete at the same position.

4. The strain at the extreme concrete compressive fibre was taken to be .003. At other positions the strain in the concrete was assumed to be directly proportional to the distance from the neutral axis.

5. The stress in the reinforcing was calculated using a modulus of elasticity of 29×10^6 lb/sq.in. Where the strain exceeded the strain corresponding to the yield stress, fy, the stress was assumed to be independent of strain and equal to fy.

6. The neutral axis was assumed to be at a distance a/0.85 from the extreme compressive fibre, where a is the distance from the extreme compressive fibre over which the equivalent rectangular stress block is assumed to act.

The axial loads in the columns were assumed to remain constant at the values caused by vertical loads alone, for the purposes of calculating the ultimate strengths of the column sections.

The amount and distribution of the reinforcement in the members is shown in Figure 5.9.

The computer program "DYNEPRES" considers only bending deformation so it was necessary at this stage to neglect shear deformation, joint size and axial deformation; however a second computer program "JOINT" was written which considered the first two parameters and the results from this program are discussed in Chapter 6.

The frame was analysed using yield moments calculated on the basis outlined above and with 5% of critical damping. It was also analysed with yield moments assigned using a load factor of 1.25 for the beam yield moments and a factor of 1.50 for the column yield moments based on the moments given by NZSS 1900 code lateral, dead and seismic live loads. A third pattern of yield moments was assigned using a load factor of 1.25 for the beams and columns, as detailed in the ACI code and also in the M.O.W. code "Design of Public Buildings". The second pattern was studied to see if the extra load factor on the column yield moment would be sufficient to prevent the formation of plastic hinges in the columns.

5.7 RESULTS

The storey displacements when the top storey has reached its maximum displacement are plotted in Figure 5.10. They are compared with the maximum response assuming completely elastic behaviour, with the corresponding amount of critical damping. The variation of the lateral displacements of the floors of the frame with time is shown in Figure 5.11 for the second pattern of yield moments. This response should be compared with that shown in Figure 5.8 assuming elastic behaviour. The character of the elasto-plastic response is essentially the same as the elastic response, but there is a slight lengthening of the periodicity of motion.

The magnitude of the maximum displacement is greatly reduced. The response is essentially identical up to 1.80 seconds but the peak occurring at 1.90 seconds is larger in the elasto-plastic case when first yielding occurs because the frame becomes momentarily more flexible. The next two peaks which give the maximum response for the elastic frame are much less for the elasto-plasti frame because the frame stores less elastic strain energy which is converted to kinetic energy when the motion of the frame reverses. The maximum response occurs with the elasto-plastic frame when the first plastic deformation is occurring whereas with the elastic frame it occurs on the third major peak of response after the kinetic and strain energy stored in the structure have been built up.

In all three yield moment patterns the top storey displacement is less when elasto-plastic action is considered. When the yield moments are reduced from the first to the second and third cases considered, the top storey displacement is reduced but the tendency for the lower storeys to deflect proportionally more than the top storey is accentuated, as shown in Figure 5.10.

The magnitude of the yield moments assumed and the required ductility of the various sections are shown in Figure 5.12 for the three patterns of yield moment.

The yield strengths of the columns are reduced progressively in magnitude, and relative to the beam strengths from cases one to three. The effect of this is to increase the amount of yielding in the columns and to reduce the yielding in the beams as shown in Figure 5.12 and also in Figure 5.13, which compares the required ductility to the ratio of moment assuming elastic behaviour to the yield moment for the same section.

Figure 5.13 also shows how the greatest ductility is required in the member with the highest moment ratio. For the first yield moment pattern this is the third floor beam, for the second and third patterns it is the base of the bottom column. It is clear that there is a tendency for the plastic deformations to concentrate in the relatively weakest members.

The variation of the moment at the base of the bottom column with time is shown in Figure 5.14 for the three cases considered with the growth of plastic deformation shown under each curve. The first case does not reach yield. The effect of the plastic action is to cut off the peak of the moment curve giving a distinctive flat crest to the curve. The plastic action does not appear to alter the shape of the moment curve either side of the crest. This is shown with the second and third cases for the base column moment where the yield moment is reduced from 7.70×10^6 in lb to 6.50×10^6 in lb, and this merely cuts off the top of the moment curve earlier, the shape being unaltered. Because plastic action is present for a longer time the plastic deformation is larger as shown in Figure 5.13.

The variation of moment at the right end of the third floor beam with time is shown in Figure 5.15 for the three cases considered; with the change of plastic deformation shown under each curve. This figure shows how the yielding of other sections in the second and third cases limits the response of the structure and hence the required ductility even though the yield moment is less.

The moment curves in Figures 5.14 and 5.15 do not have zero moment at zero time because the initial moments were assumed to be those caused by a vertical code dead and seismic live loading.

The analysis of the structure with a program which considers the effects of joint size and shear deformation is dealt with after the development of the program has been outlined. It should be noted that a relatively crude attempt was made to compensate for the effects of joint size with the first program in that the yield moments were increased to allow for the fact that the hinge would actually form on a line with the adjacent member face and not at the centre line intersection and hence the member would not be called on to resist the moment at the centre line but only that at the adjacent member face. The yield moments calculated from the reinforcement detailed, that is Pattern 1, were increased in proportion to the ratio of the centre line and clear lengths. This meant that the yield moments of the column sections were increased relatively more than the beams. Patterns 2 and 3 were derived from the moments at the centre line intersections caused by code loading.

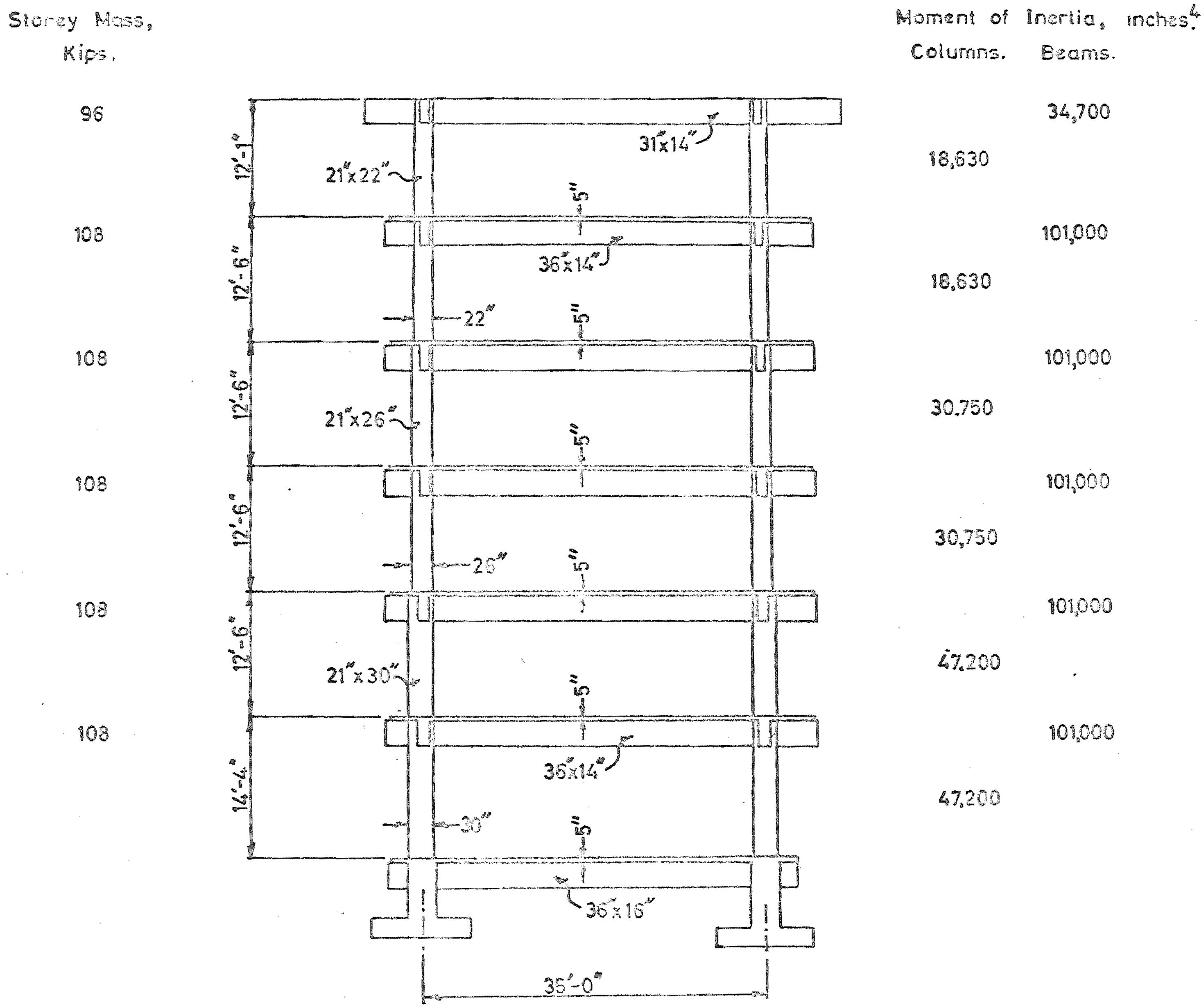

Storey Mass, Kips.
96
108
108
108
108
108
Moment of Inertia, inches^4.
Columns.
Beams.
34,700
18,630
101,000
18,630
101,000
30,750
101,000
30,750
101,000
47,200
101,000
47,200
12'-1"
12'-6"
12'-6"
12'-6"
12'-6"
14'-4"
31"x14"
21"x22"
5"
36"x14"
22"
21"x26"
26"
21"x30"
36"x14"
30"
36"x16"
35'-0"

FIGURE 5·6

PREDICTED ELASTIC RESPONSE. CODE LOADING.

Moments in inch pounds reduced by 10^6.

Forces in kips.

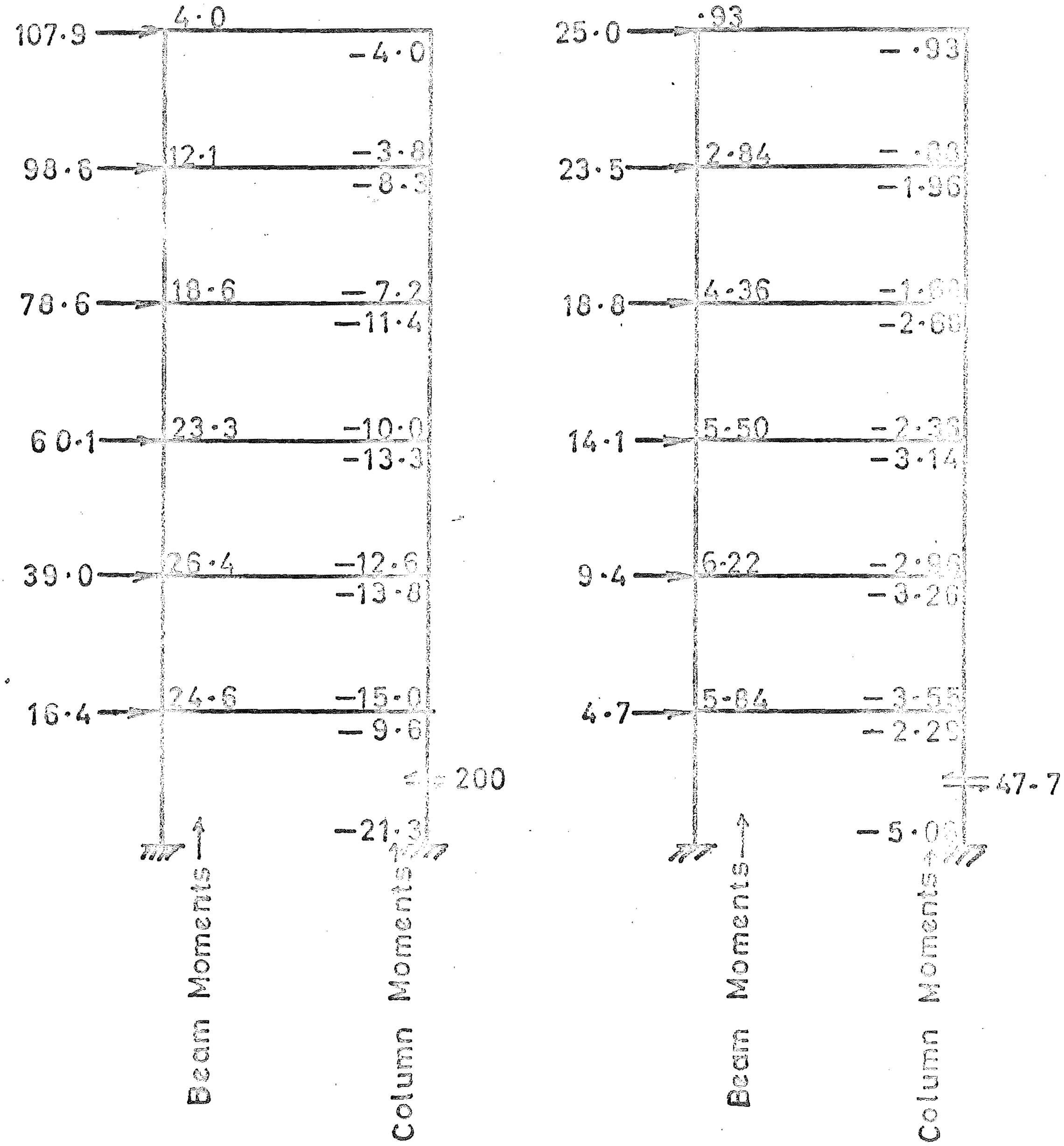


FIGURE 5·7

TABLE 5.2

NORMAL MODE PROPERTIES

Mode 1

Frequency = 1.89 c.p.s. Period = .527 secs.

Amplification Factor = .79

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	3.611	127.3
.895	3.233	255.5
.736	2.657	360.9
.548	1.980	439.5
.330	1.191	486.7
.126	.456	504.8

Mode 2

Frequency = 5.62 c.p.s. Period = .178 secs.

Amplification Factor = .79

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	-.1531	-47.5
.272	-.0416	-62.0
-.486	.0744	-36.0
-.863	.1321	10.0
-.776	.1189	51.5
-.359	.0549	70.7

TABLE 5.2 Continued

Mode 3

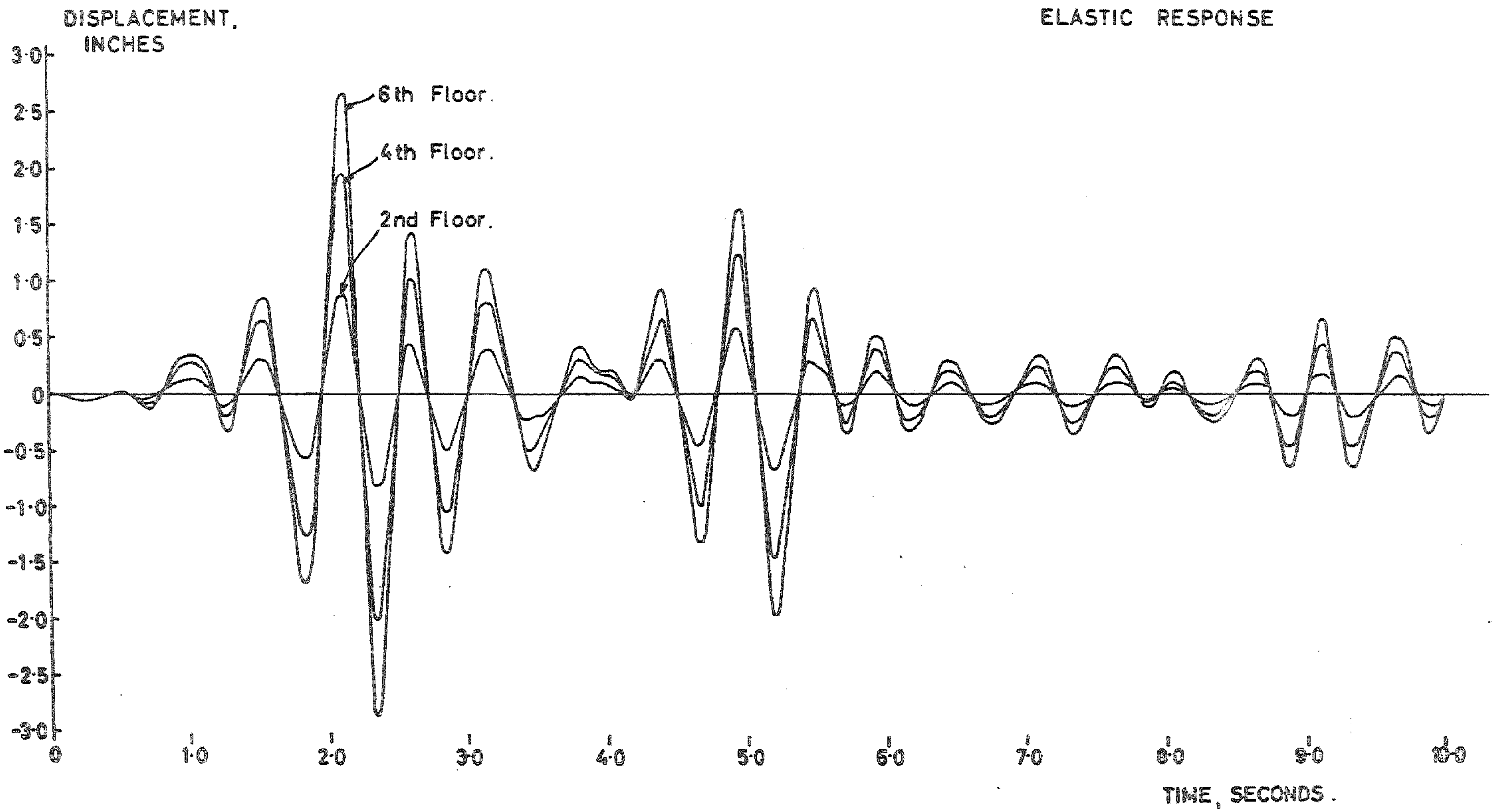
Frequency = 9.95 c.p.s. Period = .101 secs.

Amplification Factor = .67

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	.0255	24.7
-.736	-.0187	4.3
-.995	-.0253	-23.4
.121	.0031	-20.0
1.046	.0266	9.0
.714	.0182	28.9

PREDICTED ELASTIC RESPONSE

Displacement inches	Shears Kips	Forces Kips
2.87	109.2	109.2
2.57	208.8	99.6
2.11	288.4	79.6
1.58	349.3	60.8
.95	388.6	39.4
.37	405.2	16.6



DISPLACEMENT, INCHES
ELASTIC RESPONSE
6th Floor.
4th Floor.
2nd Floor.
3·0
2·5
2·0
1·5
1·0
0·5
0
-0·5
-1·0
-1·5
-2·0
-2·5
-3·0
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 5·8

DETAILS OF REINFORCEMENT.

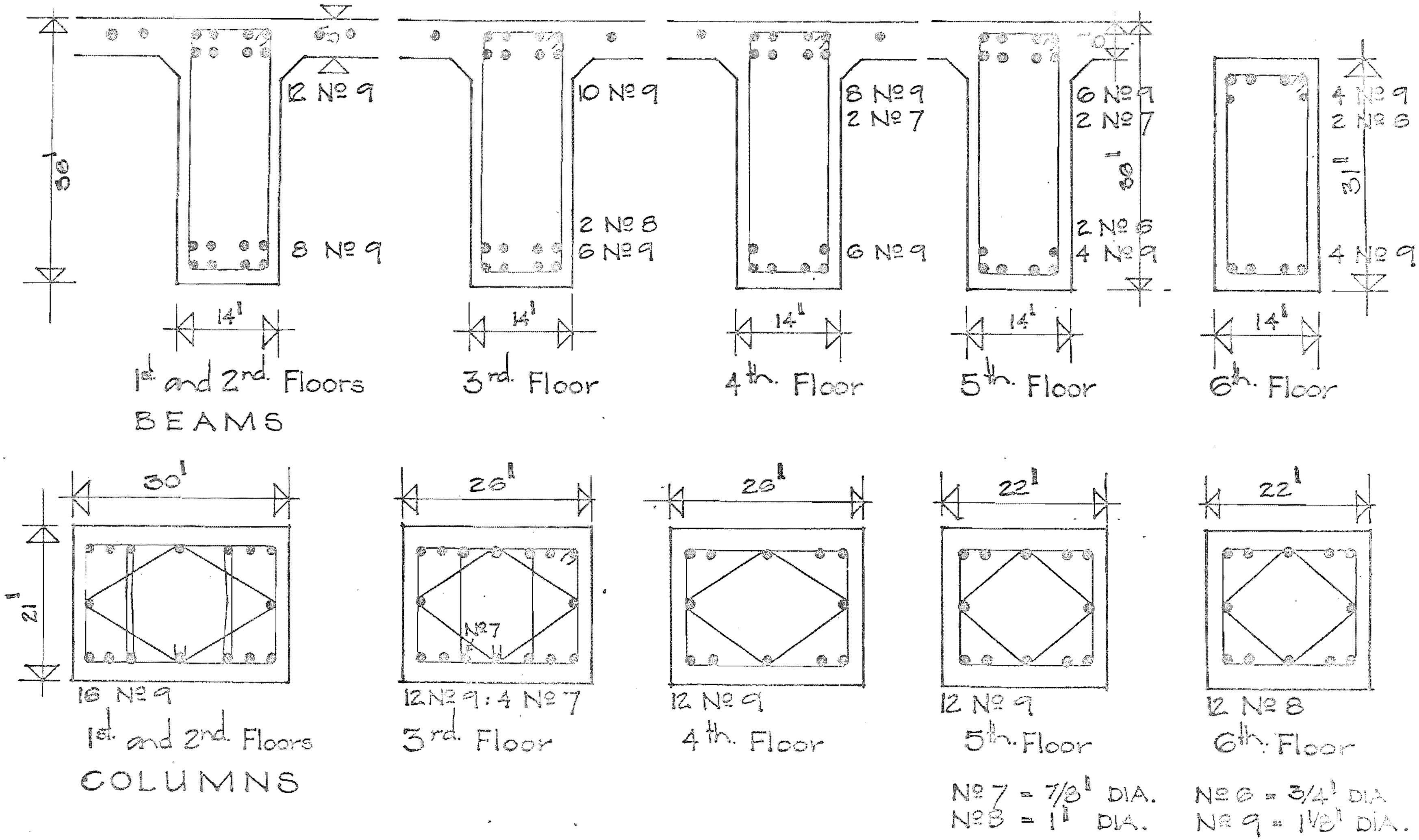


FIGURE 5.9

COMPARISON OF DISPLACEMENTS.

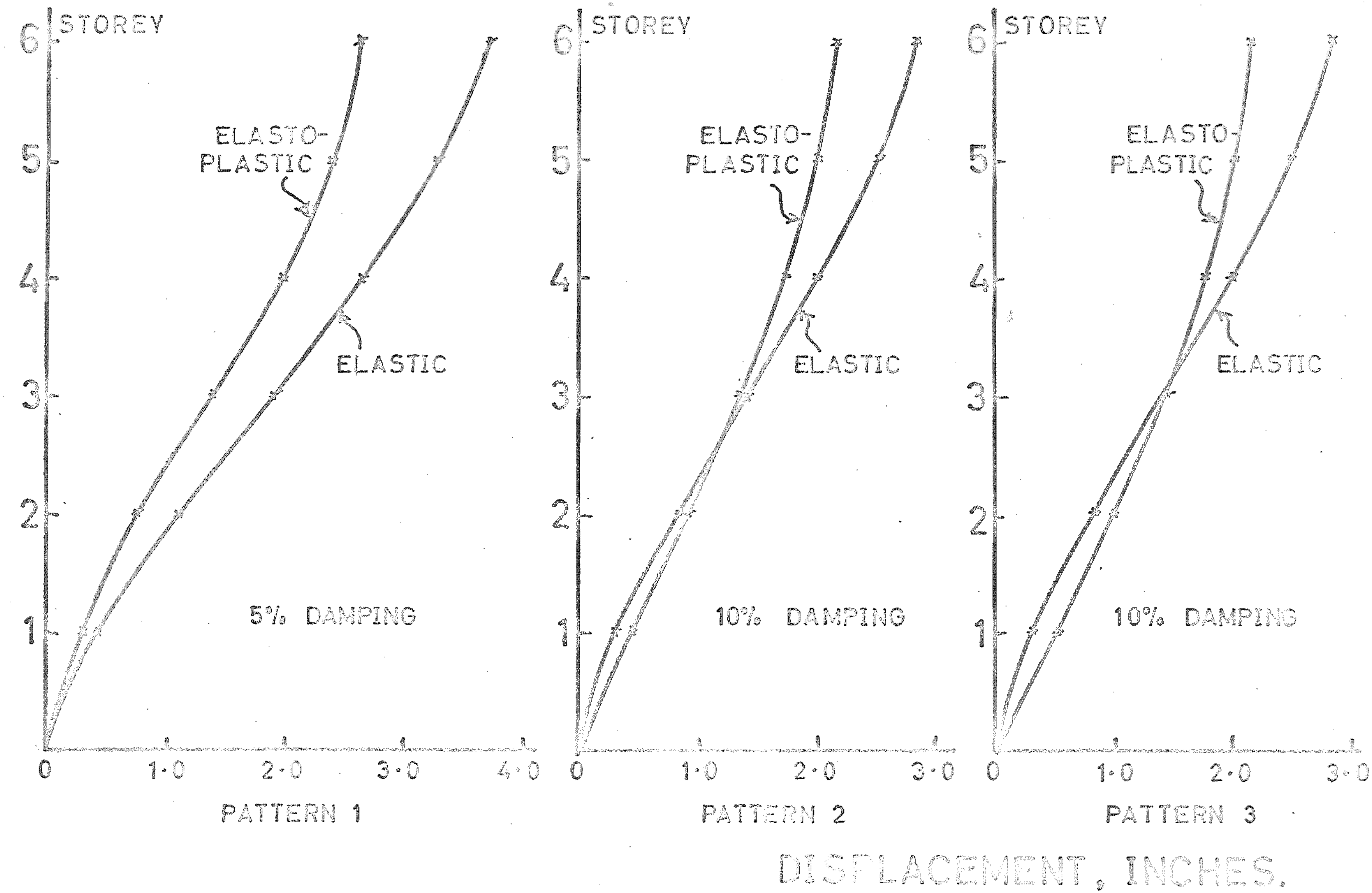


FIGURE 5·10

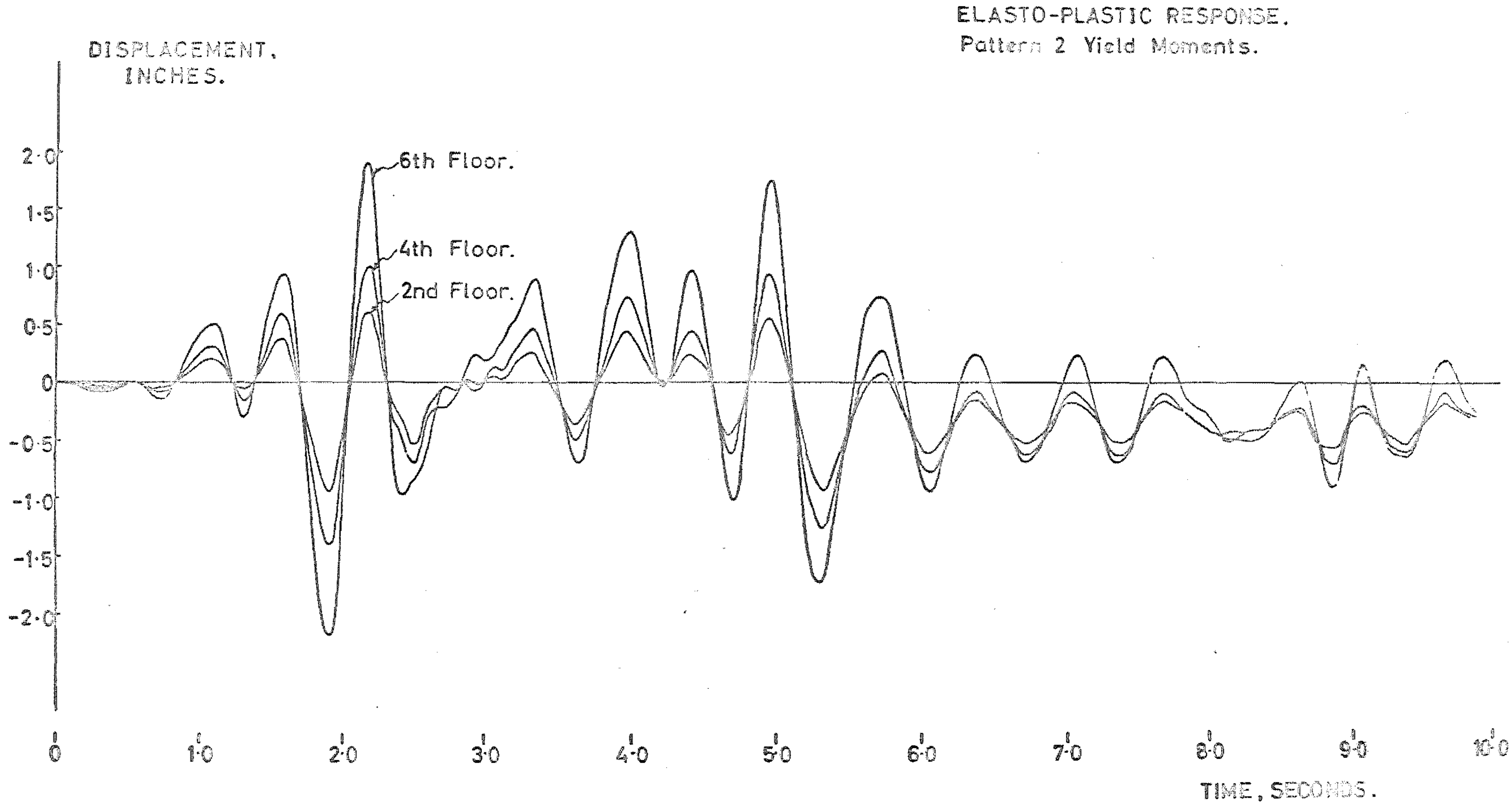
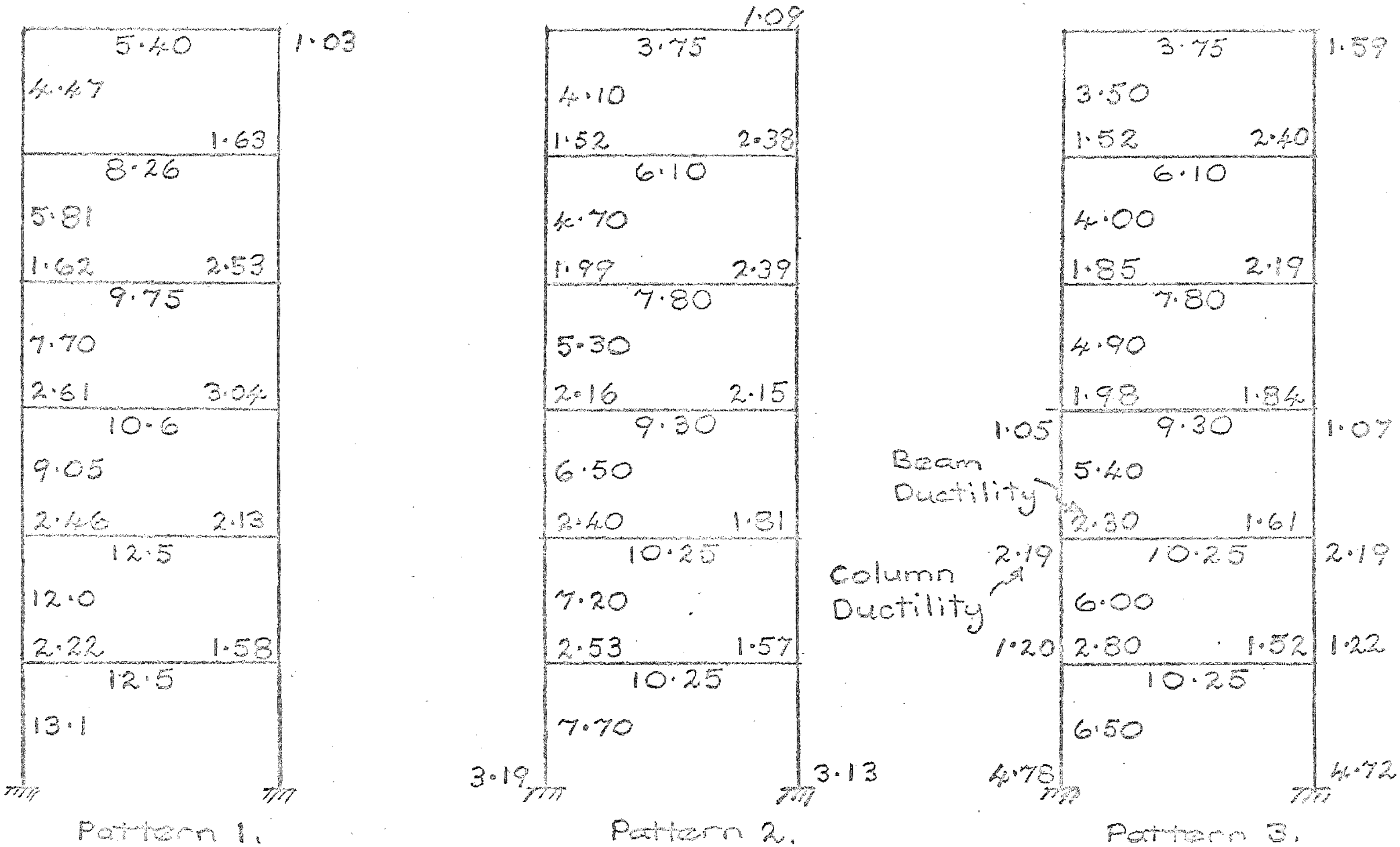

ELASTO-PLASTIC RESPONSE.
Pattern 2 Yield Moments.
DISPLACEMENT, INCHES.
6th Floor.
4th Floor.
2nd Floor.
2·0
1·5
1·0
0·5
0
-0·5
-1·0
-1·5
-2·0
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 5·11

MEMBER YIELD MOMENTS AND DUCTILITIES.



Yield Moments in inch pounds reduced by 10^6, at member centres.
Ductilities at member ends; where not marked no yielding occurred.
Both columns on a floor have the same yield moment.

FIGURE 5·12

DUCTILITY COMPARED WITH MOMENT RATIO

BEAM MEMBERS except where indicated.

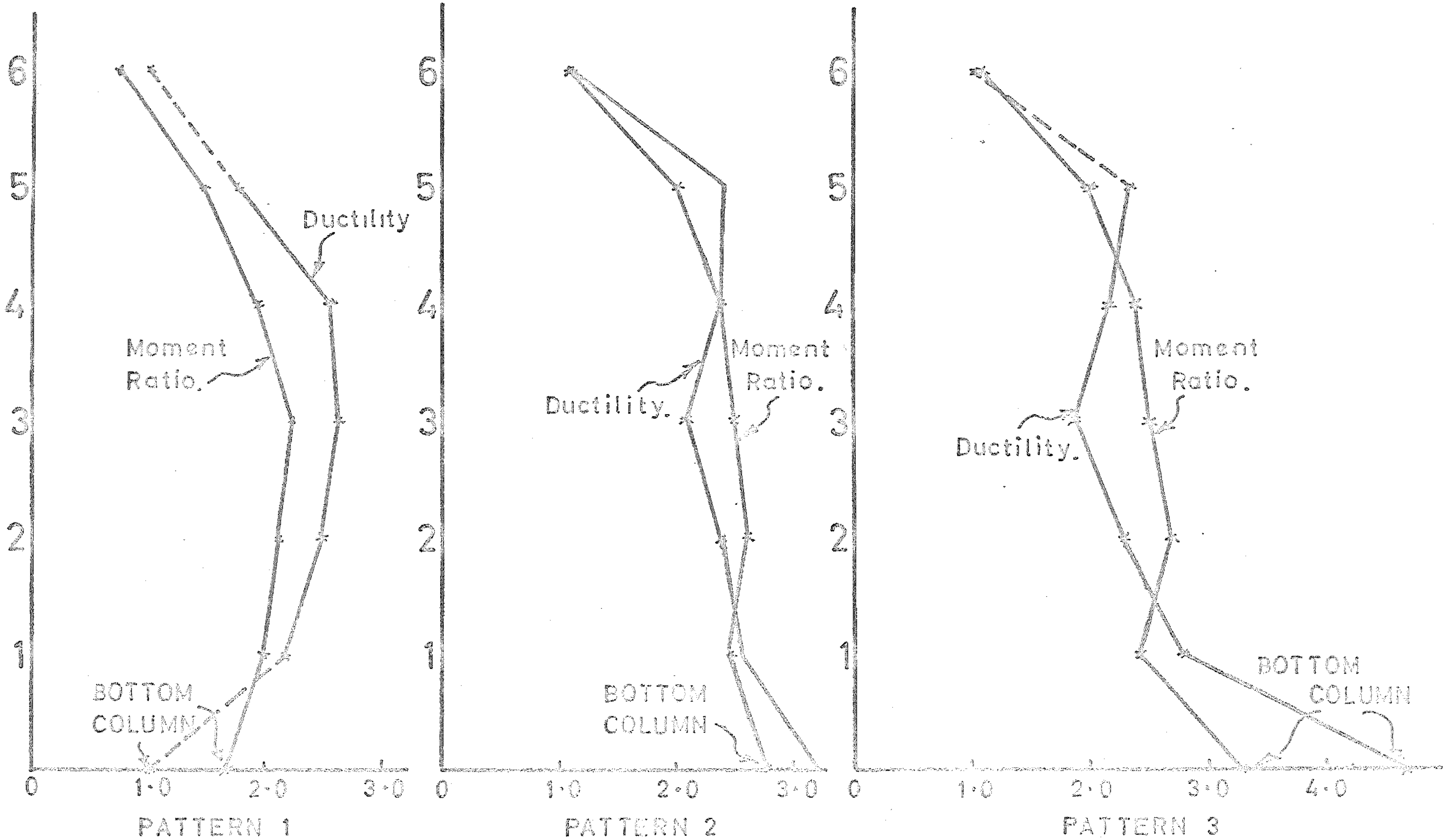


FIGURE [illegible]

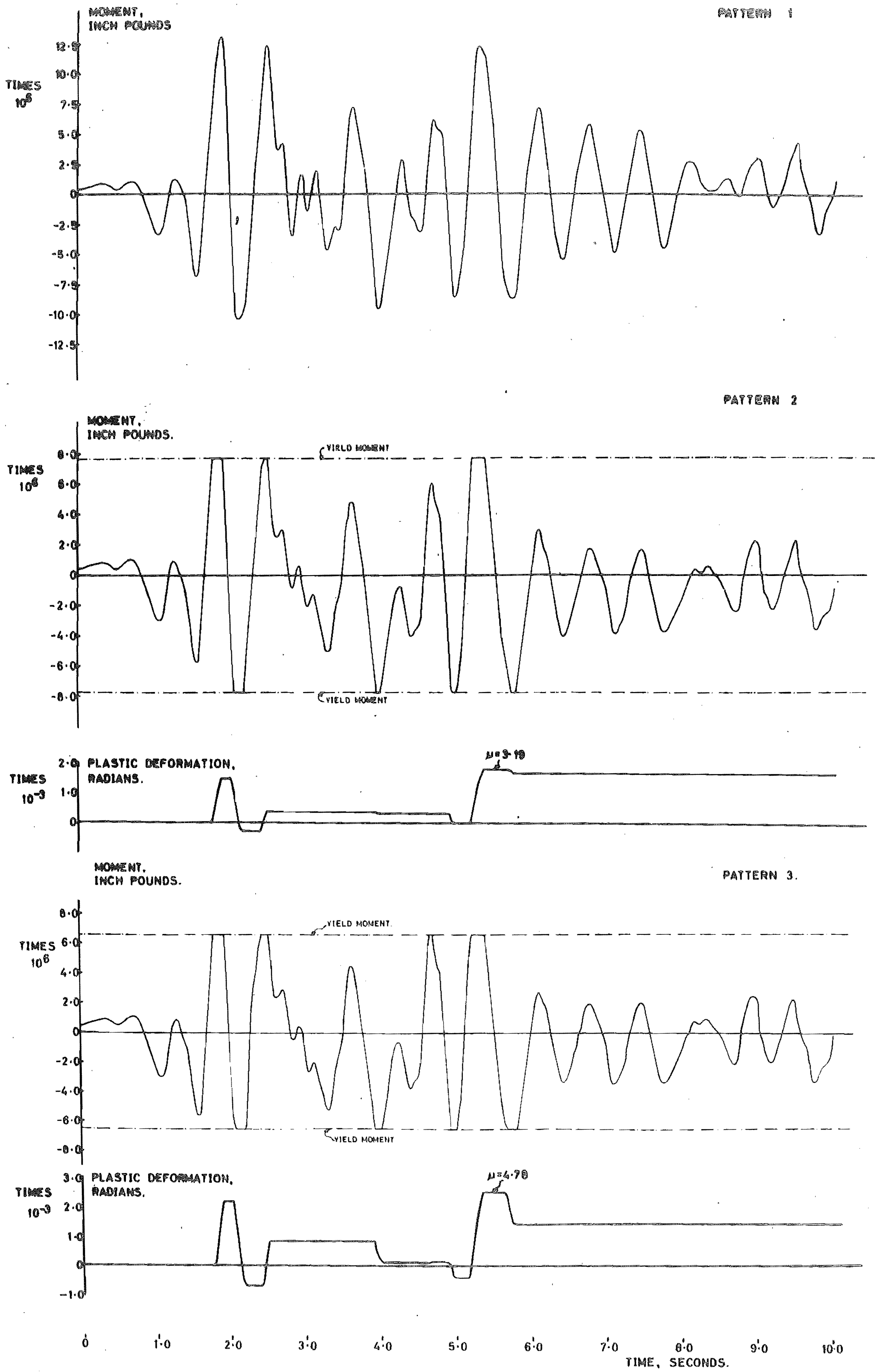
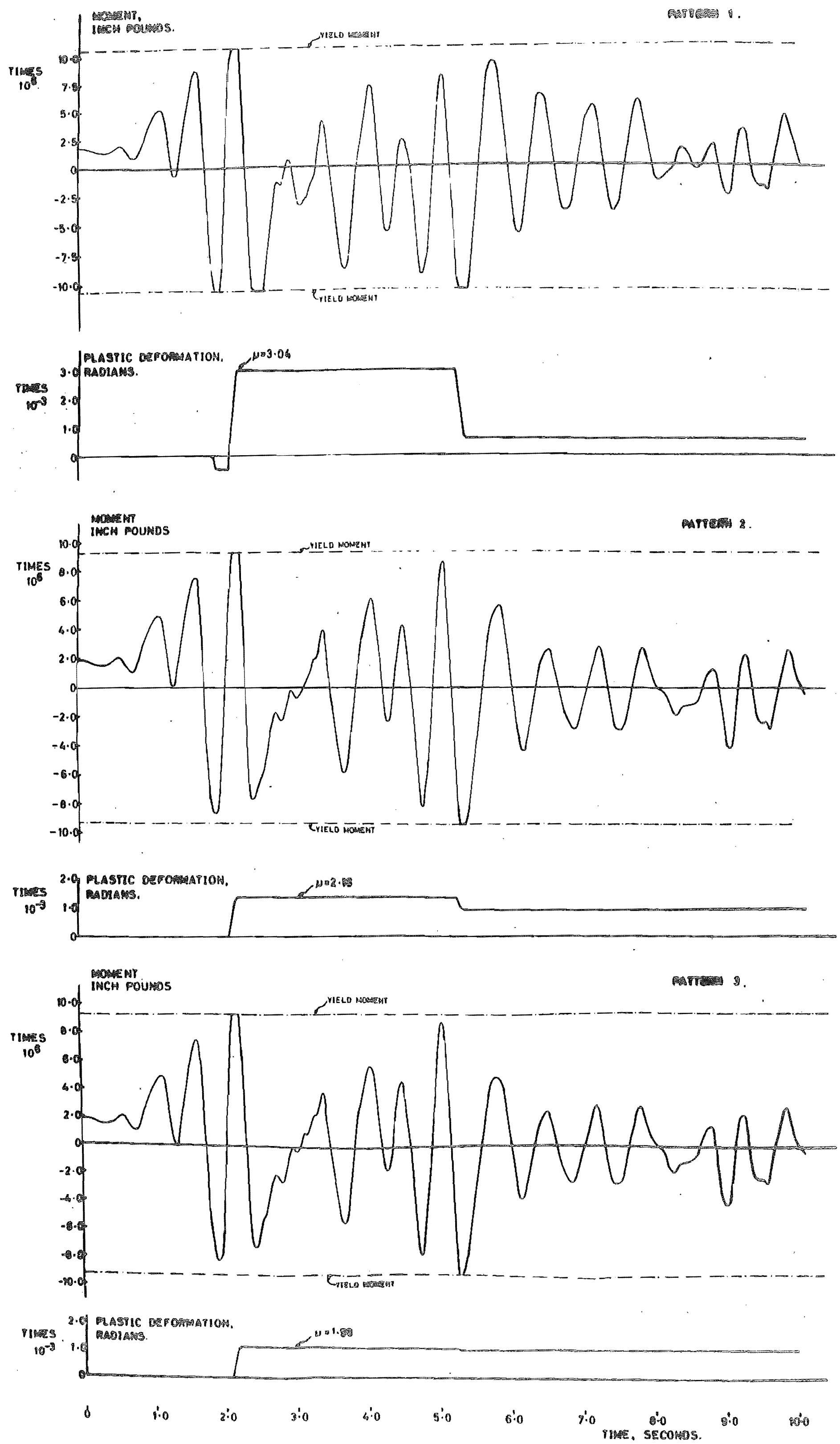

MOMENT, INCH POUNDS
PATTERN 1
TIMES 10^6
12.5
10.0
7.5
5.0
2.5
0
-2.5
-5.0
-7.5
-10.0
-12.5
MOMENT, INCH POUNDS.
PATTERN 2
YIELD MOMENT
TIMES 10^6
8.0
6.0
4.0
2.0
0
-2.0
-4.0
-6.0
-8.0
YIELD MOMENT
PLASTIC DEFORMATION, RADIANS.
TIMES 10^{-3}
2.0
1.0
0
μ=3.19
MOMENT, INCH POUNDS.
PATTERN 3.
YIELD MOMENT
TIMES 10^6
8.0
6.0
4.0
2.0
0
-2.0
-4.0
-6.0
-8.0
YIELD MOMENT
PLASTIC DEFORMATION, RADIANS.
TIMES 10^{-3}
3.0
2.0
1.0
0
-1.0
μ=4.78
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
TIME, SECONDS.

FIGURE 5.14



MOMENT, INCH POUNDS.
PATTERN 1.
YIELD MOMENT
TIMES 10^6
PLASTIC DEFORMATION, RADIANS.
μ=3·04
TIMES 10^{-3}
MOMENT INCH POUNDS
PATTERN 2.
PLASTIC DEFORMATION, RADIANS.
MOMENT INCH POUNDS
PATTERN 3.
PLASTIC DEFORMATION, RADIANS.
TIME, SECONDS.

FIGURE 5·15

CHAPTER SIX

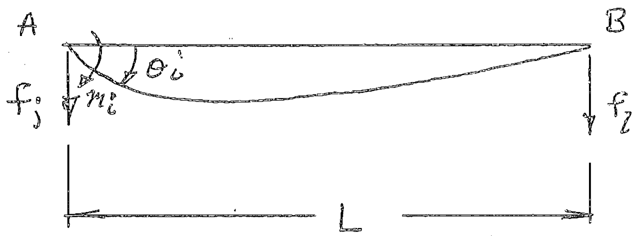
INCLUSION OF THE EFFECTS OF JOINT SIZE AND SHEAR DEFORMATION

6.1 DERIVATION OF MEMBER STIFFNESS RELATIONSHIPS

The effects of joint size and shear deformation are significant with many frames and the dynamic elasto-plastic response program "DYNEPRES" was modified to take account of them.

Stiffness matrices have already been derived for beam and column members taking joint size and shear deformations into account, when the member remains elastic, in Chapter 2. The stiffness matrices taking these effects into account when the member forms plastic hinges at either or both ends are now derived.

Consider a simple beam AB, with a moment m_i applied to end A:



Then considering bending and shear deformations:

$$\theta_i = \frac{L}{3EI} m_i + \frac{1}{\bar{A}G} \frac{m_i}{L}$$

where E, I, L, $\bar{A}$ and G are as previously defined.

Then

$$m_i = \frac{1}{\frac{L}{3EI} + \frac{1}{\bar{A}GL}} \quad \theta_i$$

$$= \frac{\frac{3EI}{L}}{1 + \beta/2} \quad \theta_i$$

where $\beta = \frac{EI}{L^2\bar{A}G}$

and $f_j = -f_l = \frac{3EI}{L^2} \frac{1.0}{1.0 + \beta/2} \theta_i$

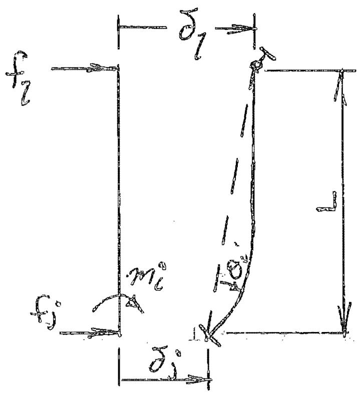
Then for a beam member:

$$\{m_i\} = [r]\{\theta_i\}$$

where

$$r = \frac{3EI}{L} \frac{1.0}{1.0 + \beta/2}$$

and for a column member:



$$\begin{Bmatrix} m_i \\ f_j \\ f_l \end{Bmatrix} = \begin{bmatrix} r & \frac{r}{L} & -\frac{r}{L} \\ \frac{r}{L} & \frac{r}{L^2} & -\frac{r}{L^2} \\ -\frac{r}{L} & -\frac{r}{L^2} & \frac{r}{L^2} \end{bmatrix} \begin{Bmatrix} \theta_i \\ \delta_j \\ \delta_l \end{Bmatrix}$$

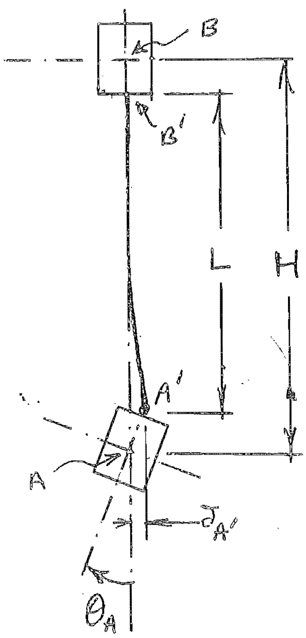
Making allowance for the sway of the column chord.

The effects of joint size must now be considered. As before, the joint is considered to be a completely rigid block. The stiffness coefficients for a member with rigid end blocks are found by calculating the actions at the centre line intersection. The member stiffness matrices derived above considering bending and shear deformations are used to calculate the actions at the junction with the joint block, and the actions at the centre line intersections found by considering the equilibrium of the rigid blocks.

Consider a column member AB with the following deformations imposed:

(Taking clockwise moments and rotations as positive, and forces and displacements from left to right as positive)

1. θ_A at A



Then $\theta_{A'} = \theta_A$

and $\delta_{A'} = \frac{H-L}{2}\theta_A$

Now $M_{A'} = 0$

and $f_A = f_{A'} = \frac{r}{L^2}\frac{H-L}{2}\theta_A$

and $f_B = f_{B'} = -f_{A'}$

and $M_{B'} = \frac{r}{L}\frac{H-L}{2}\cdot\theta_A$

$$M_B = M_{B'} - \frac{H-L}{2} f_{B'}$$

$$= \frac{r}{L} \frac{H-L}{2} \theta_A + \left(\frac{H-L}{2}\right)^2 \frac{r}{L^2} \cdot \theta_A$$

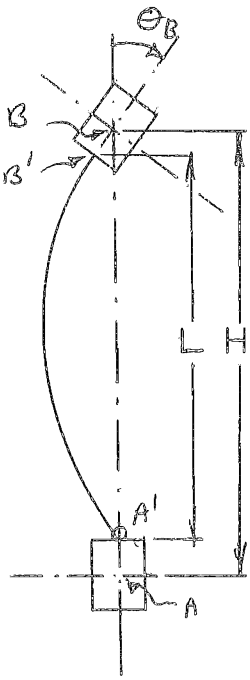
$$= \frac{r}{4L^2} (H^2-L^2)\, \theta_A$$

and $$M_A = M_{A'} + \frac{H-L}{2} f_{A'}$$

$$= 0 + \frac{r}{4L^2} (H-L)^2\, \theta_A$$

$$= \frac{r}{4L^2} (H-L)^2\, \theta_A$$

2. θ_B at B



$$\theta_{B'} = \theta_B$$

$$\delta_{B'} = -\left(\frac{H-L}{2}\right) \theta_B$$

$$M_{A'} = 0$$

and $$M_{B'} = r\,\theta_B + \frac{r}{L} \frac{H-L}{2} \theta_B$$

$$= r \frac{H+L}{2L} \theta_B$$

$$f_B = f_{B'} = -\frac{r}{L}\theta_B - \frac{r}{L^2} \frac{H-L}{2} \theta_B$$

$$= -r\,\frac{H+L}{2L^2}\,\theta_B$$

$$f_A = -f_{B'} = r\,\frac{H+L}{2L^2}\,\theta_B$$

$$M_B = M_{B'} - \frac{H-L}{2}\,f_B$$

$$= r\,\frac{H+L}{2L}\theta_B + r\,\frac{H-L}{2}\,\frac{H+L}{2L^2}\,\theta_B$$

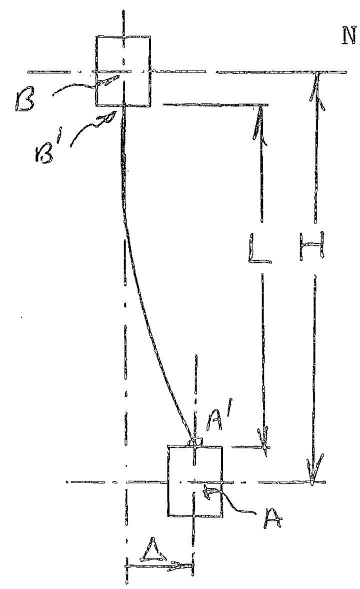
$$= r\,\frac{(H+L)^2}{4L^2}\cdot\theta_B$$

$$M_A = M_{A'} + \frac{H-L}{2}\,f_A$$

$$= 0 + r\,\frac{H-L}{2}\,\frac{H+L}{2L^2}\,\theta_B$$

$$= \frac{r}{4L^2}\,(H^2-L^2)\,\theta_B$$

3. Δ at A



Now $\delta_{A'} = \Delta$

$$M_{A'} = 0$$

$$M_{B'} = \frac{r}{L^2}\Delta$$

$$f_{A'} = \frac{r}{L^2}\Delta$$

$$M_B = M_{B'} - f_{B'} \frac{H-L}{2}$$

$$= \frac{r}{L}\Delta + \frac{r}{L^2} \frac{H-L}{2}\Delta$$

$$M_B = \frac{r}{L} \frac{H+L}{2L}\Delta$$

$$f_A = \frac{r}{L^2}\Delta$$

$$f_B = -\frac{r}{L^2}\Delta$$

$$M_A = M_{A'} + \frac{H-L}{2} f_{A'}$$

$$= \frac{r(H-L)}{2L^2}\Delta$$

Hence the equilibrium equations for a column member AB with a hinge at A

$$\begin{Bmatrix} M_A \\ M_B \\ f_A \\ f_B \end{Bmatrix} = \begin{bmatrix} \frac{r}{4L^2}(H-L)^2 & \frac{r}{4L^2}(H^2-L^2) & \frac{r}{2L^2}(H-L) & -\frac{r}{2L^2}(H-L) \\ \frac{r}{4L^2}(H^2-L^2) & \frac{r}{4L^2}(H+L)^2 & \frac{r}{2L^2}(H+L) & -\frac{r}{2L^2}(H+L) \\ \frac{r}{2L^2}(H-L) & \frac{r}{2L^2}(H+L) & \frac{r}{L^2} & -\frac{r}{L^2} \\ -\frac{r}{2L^2}(H-L) & -\frac{r}{2L^2}(H+L) & -\frac{r}{L^2} & +\frac{r}{L^2} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \\ \delta_A \\ \delta_B \end{Bmatrix}$$

and for a beam member AB with a hinge at A

$$\begin{Bmatrix} M_A \\ \\ M_B \end{Bmatrix} = \begin{bmatrix} \frac{r}{4L^2}\,(H-L)^2 & \frac{r}{4L^2}\,(H^2-L^2) \\ \\ \frac{r}{4L^2}\,(H^2-L^2) & \frac{r}{4L^2}\,(H+L)^2 \end{bmatrix} \begin{Bmatrix} \theta_A \\ \\ \theta_B \end{Bmatrix}$$

Similarly for a column member AB with a hinge at B

$$\begin{Bmatrix} M_A \\ \\ M_B \\ \\ f_A \\ \\ f_B \end{Bmatrix} = \begin{bmatrix} \frac{r}{4L^2}\,(H+L)^2 & \frac{r}{4L^2}\,(H^2-L^2) & \frac{r(H+L)}{2L^2} & -\frac{r(H+L)}{2L^2} \\ \\ \frac{r}{4L^2}\,(H^2-L^2) & \frac{r}{4L^2}\,(H-L)^2 & \frac{r(H-L)}{2L^2} & -\frac{r}{2L^2}\,(H-L) \\ \\ \frac{r}{2L^2}\,(H+L) & \frac{r}{2L^2}\,(H-L) & \frac{r}{L^2} & -\frac{r}{L^2} \\ \\ -\frac{r}{2L^2}\,(H+L) & \frac{-r}{2L^2}\,(H-L) & -\frac{r}{L^2} & \frac{r}{L^2} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \\ \theta_B \\ \\ \delta_A \\ \\ \delta_B \end{Bmatrix}$$

and for a beam member AB with a hinge at A

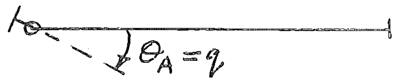
$$\begin{Bmatrix} M_A \\ \\ M_B \end{Bmatrix} = \begin{bmatrix} \frac{r}{4L^2}\,(H+L)^2 & \frac{r}{4L^2}\,(H^2-L^2) \\ \\ \frac{r}{4L^2}\,(H^2-L^2) & \frac{r}{4L^2}\,(H-L)^2 \end{bmatrix} . \begin{Bmatrix} \theta_A \\ \\ \theta_B \end{Bmatrix}$$

6.2 CALCULATION OF THE PLASTIC DEFORMATIONS

The relationships derived previously must be modified to include shear deformation and joint size effects.

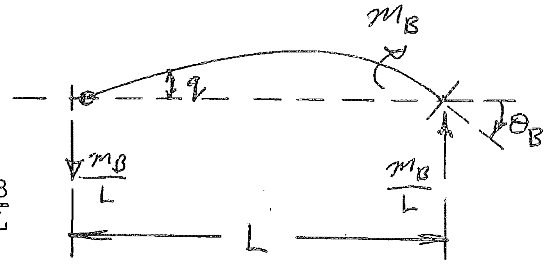
Applying each joint deformation in turn and considering bending and shear deformations only, with one hinge at A, the rotation q of the hinge may be found.

a) With θ_A imposed



$$q = \theta_A$$

b) With θ_B imposed



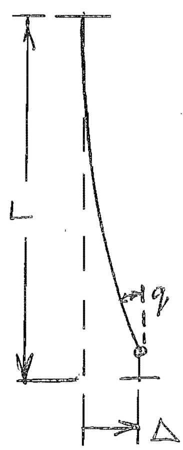
$$\theta_B = \frac{L}{3EI} M_B + \frac{1}{\bar{A}G} \frac{M_B}{L}$$

$$\theta_A = -\frac{L}{6EI} M_B + \frac{1}{\bar{A}G} \frac{M_B}{L}$$

$$\theta_A = -\frac{1-\beta}{2+\beta} \theta_B$$ eliminating M_B

$$q = \frac{1-\beta}{2+\beta} \theta_B$$ taking a clockwise rotation of joint A relative to the member as giving a positive kink.

c) With Δ imposed

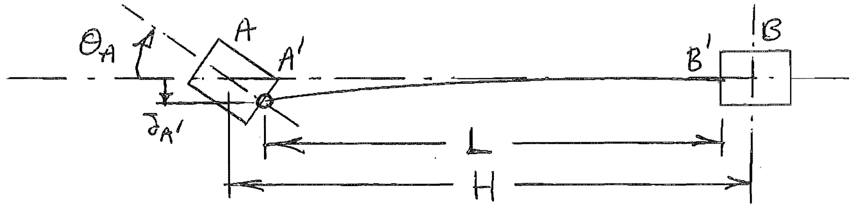


$$q = \frac{1-\beta}{2+\beta} \frac{\Delta}{L} + \frac{\Delta}{L}$$

$$= \frac{3}{2+\beta} \frac{\Delta}{L}$$

Now including joint size, together with bending and shear deformation, with one hinge at A, the rotation of the hinge may be found:

a) With θ_A imposed

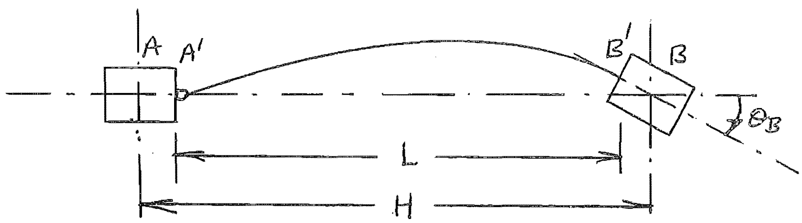


$$\delta_{A'} = \frac{H-L}{2}\,\theta_A$$

$$q = \theta_A + \frac{3}{2+\beta}\,\frac{H-L}{2L}\,\theta_A$$

$$= \left(1 + \frac{1.5}{2+\beta}\,\frac{H-L}{L}\right)\theta_A$$

b) With θ_B imposed



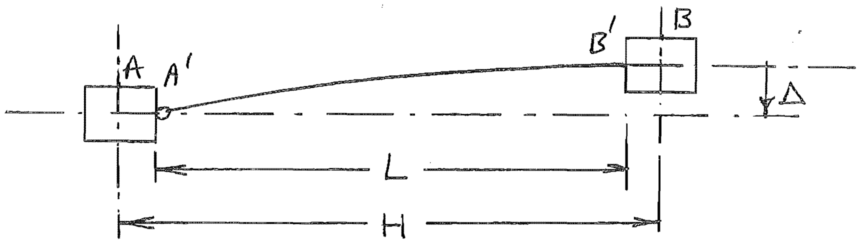
$$\delta_{B'} = \frac{H-L}{2}\,\theta_B$$

$$q = \frac{1-\beta}{2+\beta}\,\theta_B + \frac{3}{2+\beta}\,\frac{\delta}{L}$$

$$= \frac{1-\beta}{2+\beta}\,\theta_B + \frac{3}{2+\beta}\,\frac{H-L}{2L}\,\theta_B$$

$$= \left(\frac{1-\beta}{2+\beta} + \frac{1.5}{2+\beta}\,\frac{H-L}{L}\right)\cdot\theta_B$$

c) With Δ imposed

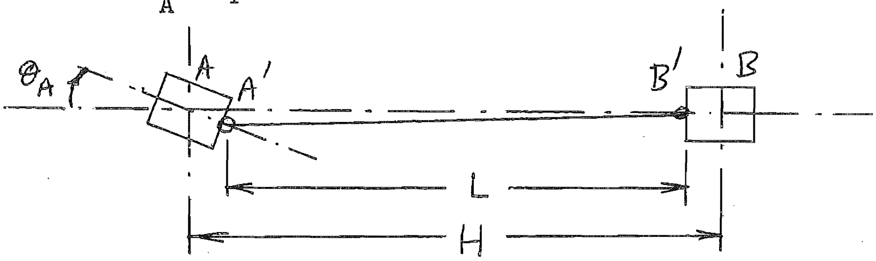


$$q = \frac{3}{2+\beta} \frac{\Delta}{L}$$

Similar relationships may be derived with one hinge at B.

With hinges at both A and B, the rotations of the hinges may be found:

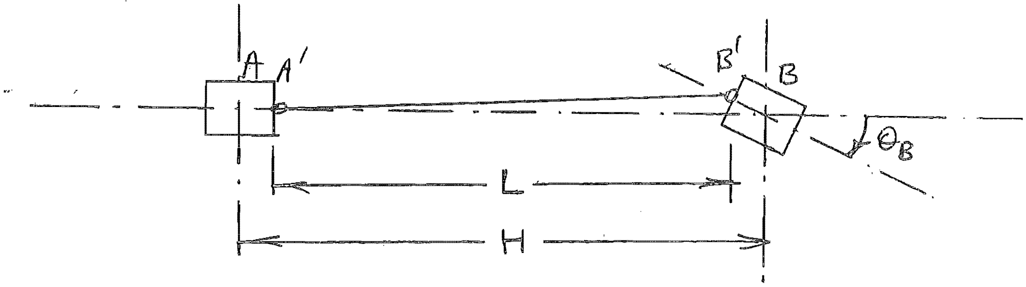
a) With θ_A imposed



$$q_A = \theta_A + \frac{H-L}{2L} \theta_A$$

$$q_B = \frac{H-L}{2L} \theta_A$$

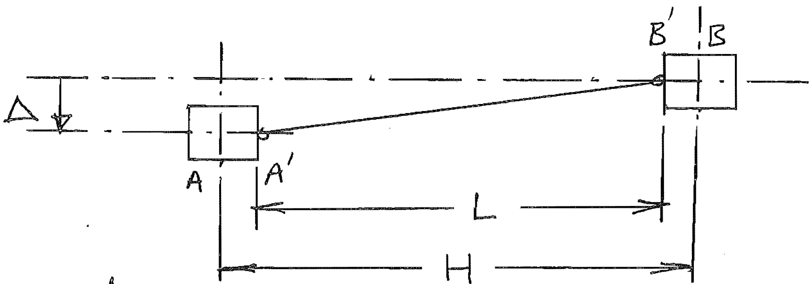
b) With θ_B imposed



$$q_A = \frac{H-L}{2L}\,\theta_B$$

$$q_B = (1.0 + \frac{H-L}{2L})\,\theta_B$$

c) With Δ imposed

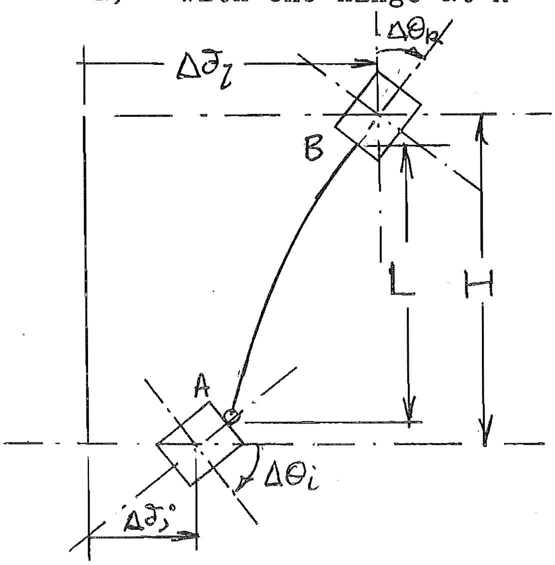


$$q_A = \frac{\Delta}{L}$$

$$q_B = \frac{\Delta}{L}$$

Hence the increase in the rotations of the hinges p_A and p_B may be calculated from the increments in the joint deformations of the frame with the following expressions:

a) With one hinge at A



$$p_A = \left(1 + \frac{1.5}{2+\beta}\ \frac{H-L}{L}\right)\Delta\theta_i + \left(\frac{1-\beta}{2+\beta} + \frac{1.5}{2+\beta}\ \frac{H-L}{L}\right)\Delta\theta_k$$

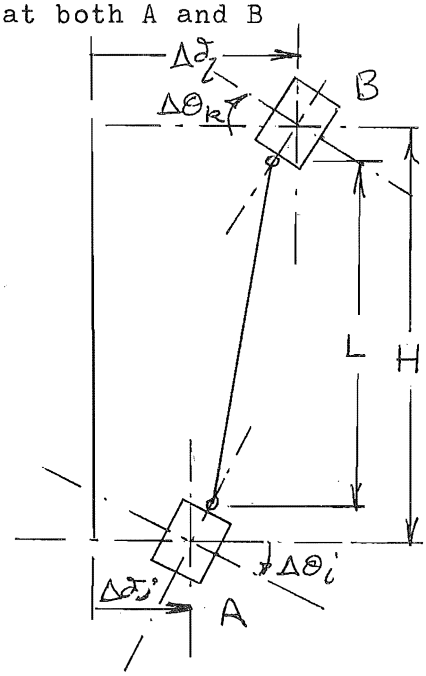
$$+ \frac{3}{2+\beta}\ \frac{\Delta\delta_j - \Delta\delta_l}{L}$$

b) With one hinge at B

$$p_B = \left(\frac{1-\beta}{2+\beta} + \frac{1.5}{2+\beta}\ \frac{H-L}{L}\right)\Delta\theta_i + \left(1 + \frac{1.5}{2+\beta}\ \frac{H-L}{L}\right)\Delta\theta_k$$

$$+ \frac{3}{2+\beta}\ \frac{\Delta\delta_j - \Delta\delta_l}{L}$$

c) With hinges at both A and B

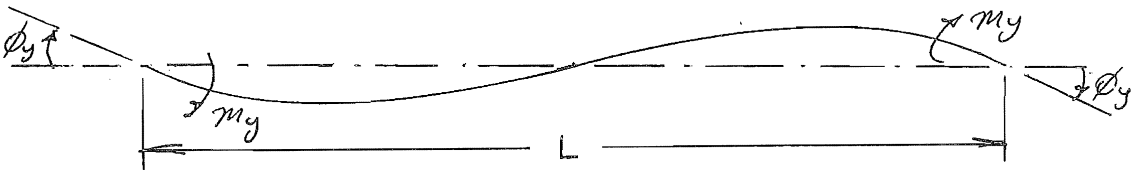


$$\Delta p_A = \left(1.0 + \frac{H-L}{2L}\right)\Delta\theta_i + \frac{H-L}{2L}\Delta\theta_k + \frac{1}{L}\ (\Delta\delta_j - \Delta\delta_l)$$

$$\Delta p_B = \frac{H-L}{2L}\Delta\theta_i + \left(1.0 + \frac{H-L}{2L}\right)\Delta\theta_k + \frac{1}{L}\ (\Delta\delta_j - \Delta\delta_l)$$

6.3 CALCULATION OF THE MEMBER DUCTILITY RATIO

The member ductility ratio is defined, as before, as the ratio of the maximum total end rotation in the member to the end rotation at the elastic limit. The elastic rotation is modified to include shear. With anti-symmetric yield moments m_y applied:



Using the equations derived in section 2.2

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \frac{L}{6EI} \begin{bmatrix} (2+\beta) & (-1+\beta) \\ (-1+\beta) & (2+\beta) \end{bmatrix} \begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix}$$

where

$$\beta = \frac{6EI}{L^2 \bar{A} G}$$

$$\phi_y = \frac{L}{6EI} (2+\beta \;\; -1+\beta)\, m_y$$

$$= \frac{m_y L}{6EI} (1+2\beta)$$

Ductility factor $\mu = 1 + \dfrac{p_{max}}{\phi_y}$

$$= 1 + \frac{6EI}{m_y L(1+2\beta)} \; p_{max}$$

where p_{max} is the maximum rotation of the plastic hinge.

The program "DYNEPRES" has been modified to incorporate the effects of joint size and shear deformation and the new program "JOINT" is given in List 11. This is dimensioned so that up to a 6-storey by $1\frac{1}{2}$ bay frame can be analysed. Larger frames were analysed with the program but it was necessary to break the program into four separate overlays because of limitations of storage with the IBM 360/44 computer at the University of Canterbury.

6.4 ANALYSIS OF BUILDING B

6.4.1 General

Building B was also analysed with the program "JOINT" which determined the dynamic elasto-plastic response considering the effects of joint size and shear deformation. The analysis of Building B detailed earlier considered bending deformations only. The yield moments and Young's Modulus were adjusted, because joint size was neglected, to give a reasonably realistic analysis. Later the computer program "JOINT" which considered joint size and shear deformation was produced, so the assumptions of yield moment and Young's Modulus were altered to what could be their actual values. It was not intended to specifically investigate the effect of joint size and shear deformation on the response, although this is implicit in the analysis. The effects of joint size are as follows: the stiffness of the frame as a whole is increased because the effective length of the members is reduced from the centre line length; secondly the position at which the plastic hinge is assumed to act is altered. The second effect means that the hinge mechanism is altered and with one hinge at the end of a member there is still some stiffness to further rotation of the joint at the same end as the hinge. When joint size is not considered the joint has no stiffness with respect to further rotation once a plastic hinge has formed at that end.

The yield moments for the program were assigned on the same basis as for the three patterns run previously except that they were calculated for the sections on a line with adjacent member face.

Pattern 1 are based on the ultimate strengths of the sections shown in Figure 5.9, assuming the ultimate strength of the concrete was 4000 lb/in^2 and the yield stress of the steel was 36,000 lb/in^2.

Pattern 2 are based on the moments caused by the NZSS 90 code (5) lateral, dead and seismic live loads using a load factor of 1.50 for the columns and 1.25 for the beams.

Pattern 3 is similar to Pattern 2 except that a load factor of 1.25 is used for both beams and columns.

Young's Modulus was assumed to be 4.0×10^6 lb/in^2 for this program. The effect of initial joint moments due to vertical loading is ignored.

6.4.2 Results of Analysis

The displaced shapes of the frame when the top storey reaches its maximum displacement are compared in Figure 6.1 with the shapes obtained assuming elastic behaviour, with the corresponding amount of critical damping, for the three patterns of yield moment. With the first pattern of yield moments the displaced shape is similar to the elastic response while the second and third patterns gave less displacement at the upper storeys. This is presumably because with the first pattern of yield moments the plastic deformations are comparatively well distributed throughout the frame whereas with the second and third patterns there is more yielding towards the bottom of the building which makes this part of the frame more flexible.

The yield moments assumed and the required ductility of the various sections is shown in Figure 6.2. The ductilities in the various members are compared in Figure 6.3 with the ratio of the maximum moment assuming elastic behaviour to the yield moment assumed for the same section. It is again evident that there is a tendency for the plastic deformations to be concentrated in the members with the highest moment ratio.

The extra load factor on the column yield moments in Pattern 2 above that used in Pattern 3 prevents yielding in all the columns except at the base of the bottom storey. The first pattern shows that low column moment ratios are not sufficient to prevent yielding in the columns. As yielding must occur somewhere, in order to dissipate energy and prevent the build-up of response, it should be restricted to the beams and the base of the bottom column by having relatively low yield moments at these sections. It is also desirable that the limit on the moment which a joint can carry is set by the sum of the yield moments of the beams at the joint and not by the sum of the yield moments of the columns. The tendency for high column ductilities to be required where the limit is set by the columns is shown in these results, particularly with Pattern 3.

COMPARISON OF DISPLACEMENTS

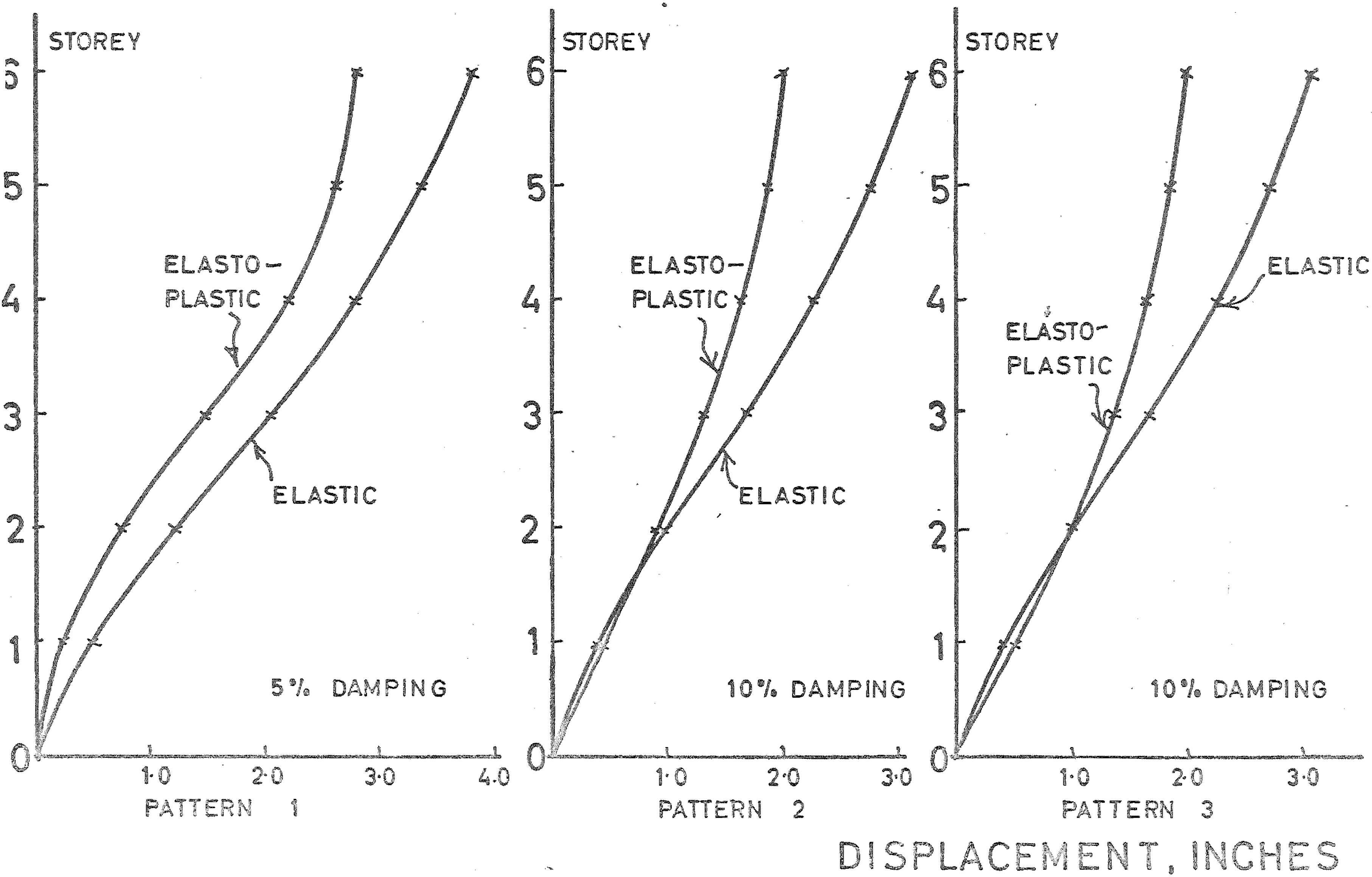
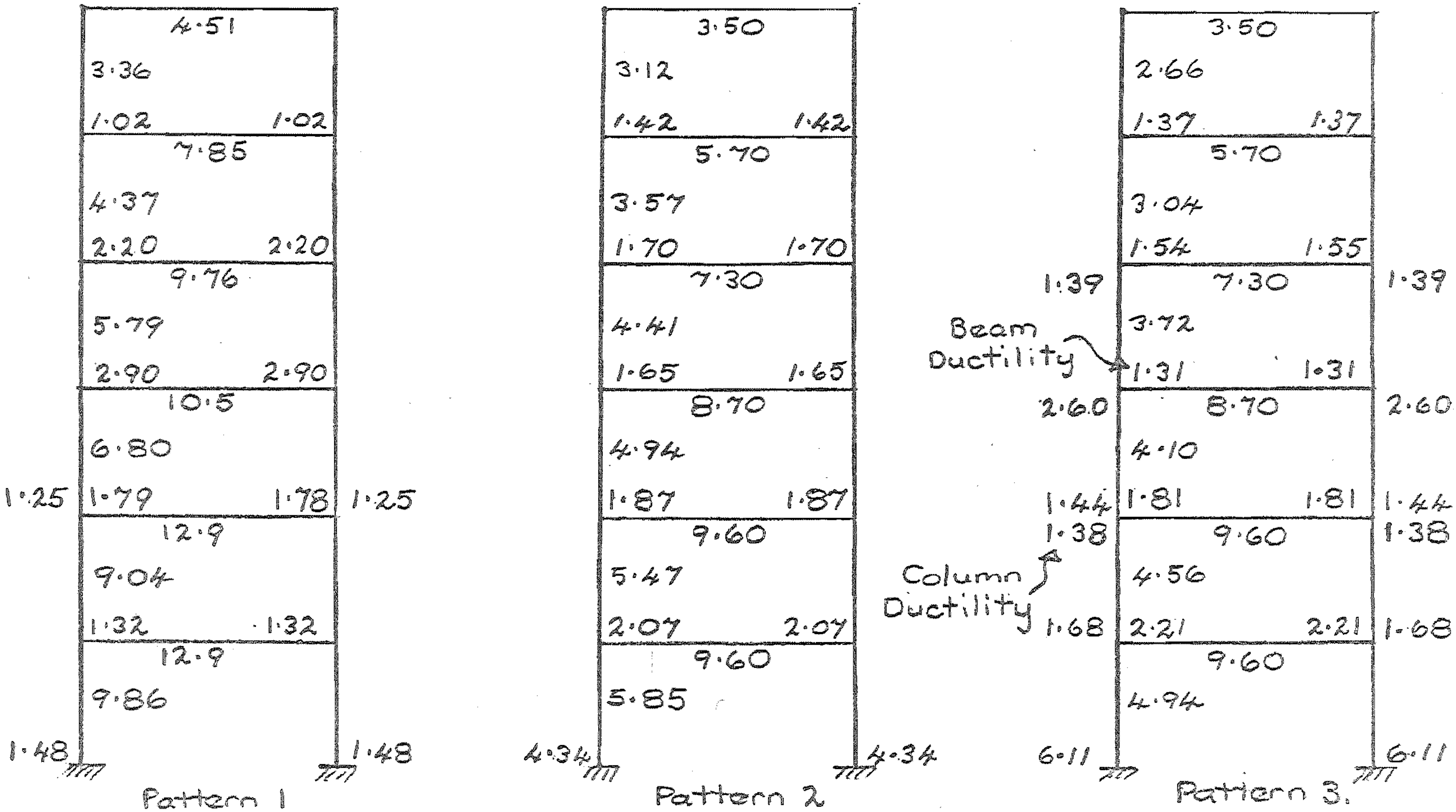


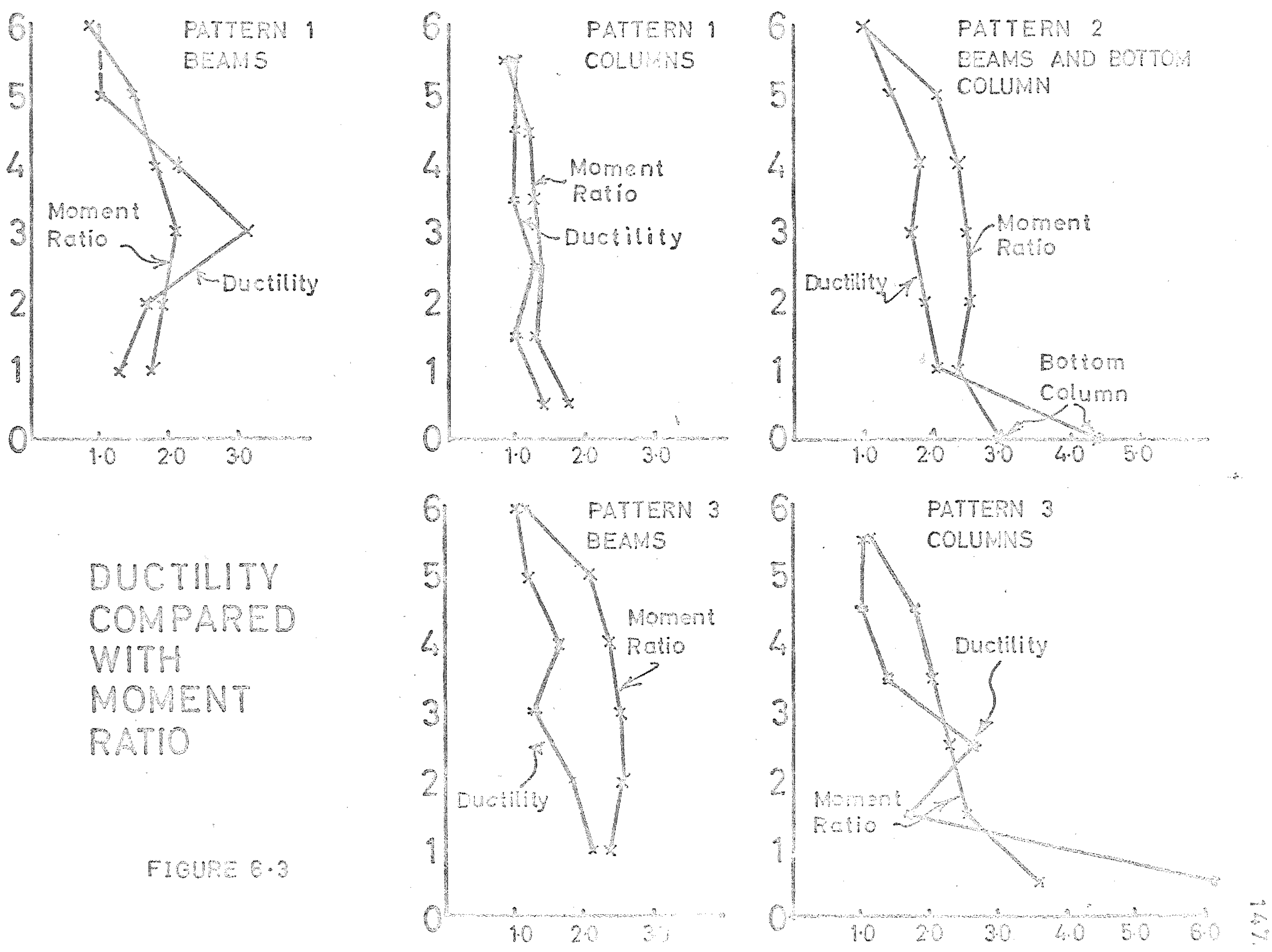
FIGURE 6·1

MEMBER YIELD MOMENTS AND DUCTILITIES.



Yield Moments in inch pounds reduced by 10^6, at member centres.
Ductilities at member ends; where not marked no yielding occurred.
Both Columns on a floor have the same yield moment.

FIGURE 6·2



DUCTILITY
COMPARED
WITH
MOMENT
RATIO

FIGURE 6·3

6.5 ANALYSIS OF BUILDING C

6.5.1 The Structure

Building C has sixteen floors, each with a floor area of approximately 5,000 square feet. A typical floor plan is shown in Figure 6.4 and the eastern elevation is shown in Figure 6.5. The reinforced concrete structure consists of a spandrel beam frame around the perimeter and flat slabs supported by light internal columns. The east and west outer faces are each supported by a two-bay and a three-bay frame which are offset 6'6" to give a northern aspect to the rear apartments. Almost all of the lateral resistance of the structure is provided by the external frames and the effect of the internal columns and floor slabs on the building's behaviour under lateral loads was neglected in this analysis. A six inch floor slab was poured above the second floor level and the sizes of the beams and columns are listed in Table 6.1.

TABLE 6.1

	Spandrel Beams	Corner Columns 8 per floor	Other External Columns
Roof down to 12th Floor (incl)	42" x 12" I = 94,200 in^4	28" x 28" I = 51,100 in^4	36" x 20" I = 77,900 in^4
12th Floor down to 7th (incl)	42" x 14" I = 105,000 in^4	28" x 28" I = 51,100 in^4	36" x 20" I = 77,900 in^4
7th Floor down to 2nd (incl)	42" x 16" I = 113,000 in^4	28" x 28" I = 51,100 in^4	36" x 20" I = 77,900 in^4

In order to keep the amount of computer time and storage required to a reasonable level it was decided to determine the elasto-plastic response of the southern portion of the eastern frame to the 1940 El Centro earthquake N-S component. It was assumed that this 13 storey 2 bay frame acted independently of the other 16 storey 3 bay frame and that the weight lumped at each floor was 216 kips. An outline of the elastic and elasto-plastic response is given in the next two sections.

6.5.2 Elastic Analysis

A normal mode analysis was carried out to enable the seismic forces to be predicted using Skinner's Response Spectrum[24]. Young's Modulus for the frame was assumed to be 4.0×10^6 lb/sq.in. The lateral flexibility matrix was determined using the program "KLAT" which considered the effects of joint size and shear deformation.

The seismic forces and displacements were predicted by taking the root mean square of the modal responses found by using the response spectrum, taking the curve for 10% critical damping. The normal properties and predicted response are listed in Table 6.2. The member actions caused by the predicted seismic forces were found by static analysis using the program "MODTDE", considering the same effects as above shown in Figure 6.6 together with the lateral forces. The member actions caused by a code type loading were also determined and are shown in Figure 6.6.

The code loads were determined using a seismic coefficient of 0.10 on the base shear and distributed in triangular fashion.

A reduction factor of roughly four is required to reduce the predicted elastic loading to that specified by the code.

6.5.3 Elasto-Plastic Analysis

The elasto-plastic response of the two bay 13 storey frame to the N-S component of the El Centro earthquake was determined using the program "JOINT" which considered the effects of shear deformation and the effect of joint size.

The ultimate moments of the beam sections were calculated using Whitney theory[25] with an equivalent rectangular stress block of average ultimate stress 0.85 fc'. The ultimate strength of concrete fc' was assumed to be 4000 lb/sq.in. The yield strength of the high tensile steel Hy 60 was assumed to be 60,000 lb/sq.in. The details of the reinforcement in the beams are given in Figure 6.7.

The ultimate moments of the column sections were derived from figures supplied by the consulting engineer, Mr. J.P. Hollings.

The damping was assumed to be 10% of critical in the first two modes.

The ultimate moments assumed for the beam and column sections are shown in Figure 6.6.

6.5.4 Results

The variation of the lateral displacement of the floors is plotted against time in Figure 6.8 assuming elastic behaviour, and in Figure 6.9 assuming elasto-plastic behaviour.

The storey displacements when the top storey has reached its maximum displacement are plotted in Figure 6.6 for the elasto-plastic response. They are compared with the displacements computed from the sum of the modal responses assuming completely elastic behaviour, and with the displacements under code loading. Shear deformation and the effect of joint size were considered in all analyses, together with 10% damping.

The top storey displacement is less when elasto-plastic behaviour is considered, but the lower storey displacements are of the same order. This is presumably because the plastic hinges form in the lower section of the building, making this more flexible and at the same time absorbing energy.

By comparing Figures 6.8 and 6.9 it can be seen that the response is identical up to the peak at 1.90 secs when the first yielding occurs. The formation of the plastic hinges makes the frame momentarily more flexible and this peak is slightly greater in the elasto-plastic response. The energy dissipated in plastic action is not returned to the frame as elastic strain energy or kinetic energy and the response peaks are all less in the elasto-plastic curves after the peak at 1.90 secs. The maximum elastic response occurs at 2.20 secs after the strain and kinetic energy have been built up, whereas the maximum elasto-plastic response occurs with the first yielding at 1.90 secs. This frame does not form any hinges in the columns except those at the base of the building. The ductility requirements of the various members are plotted in Figure 6.6. The ratios of the moments assuming elastic behaviour to the yield moments is also plotted for the corresponding members, in Figure 6.6.

Figure 6.6 shows how the plastic deformations tend to concentrate in the members with the highest moment ratio and for this frame appear to be greater near the bottom of the building for the same moment ratio. The maximum ductility is approximately 50% greater than the maximum moment ratio.

The variation of bending moment of a second floor beam is plotted against time in Figure 6.10, together with the variation of plastic deformation in the same member. The moment-rotation relationship for this member is also illustrated in Figure 4.1.

TABLE 6.2

NORMAL MODE PROPERTIES

Mode 1

Frequency = 2.07 c.p.s. Period = 0.483 secs.

Amplification Factor = 0.67

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	2.930	277
.983	2.879	550
.951	2.785	813
.905	2.652	1064
.847	2.483	1299
.777	2.277	1515
.695	2.037	1708
.603	1.768	1875
.505	1.480	2015
.403	1.180	2127
.295	.864	2208
.183	.536	2259
.072	.211	2279

Mode 2

Frequency = 6.17 c.p.s. Period = 0.162 secs.

Amplification Factor = 0.62

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	-.1134	-95.3
.853	-.0968	-176.5
.595	-.0675	-233.2
.267	-.0303	-258.7
-.084	.0095	-250.7
-.424	.0481	-210.3
-.708	.0804	-142.8
-.902	.1023	-56.9
-.979	.1110	36.3
-.933	.1058	125.1
-.773	.0877	198.8
-.520	.0591	248.4
-.214	.0243	268.8

TABLE 6.2 continued

Mode 3

Frequency = 10.41　　　Period = .096 secs.

Amplification Factor = 0.54

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	.0235	56.3
.608	.0143	90.5
.008	.0002	91.0
-.577	-.0136	58.5
-.940	-.0221	5.6
-.975	-.0229	-49.2
-.671	-.0158	-87.0
-.136	-.0032	-94.6
.434	.0102	-70.2
.846	.0199	-22.6
.979	.0230	32.5
.790	.0186	77.0
.357	.0084	97.1

PREDICTED ELASTIC RESPONSE

Displacement inches	Shears Kips	Forces Kips
1.986	199.	199.4
1.951	392.	192.2
1.887	572.	180.6
1.796	740.	167.5
1.682	894.	154.4
1.543	1035.	140.8
1.381	1161.	126.2
1.199	1271.	110.5
1.005	1366.	94.1
.802	1443.	77.1
.588	1501.	58.5
.365	1539.	37.5
.144	1554.	15.1

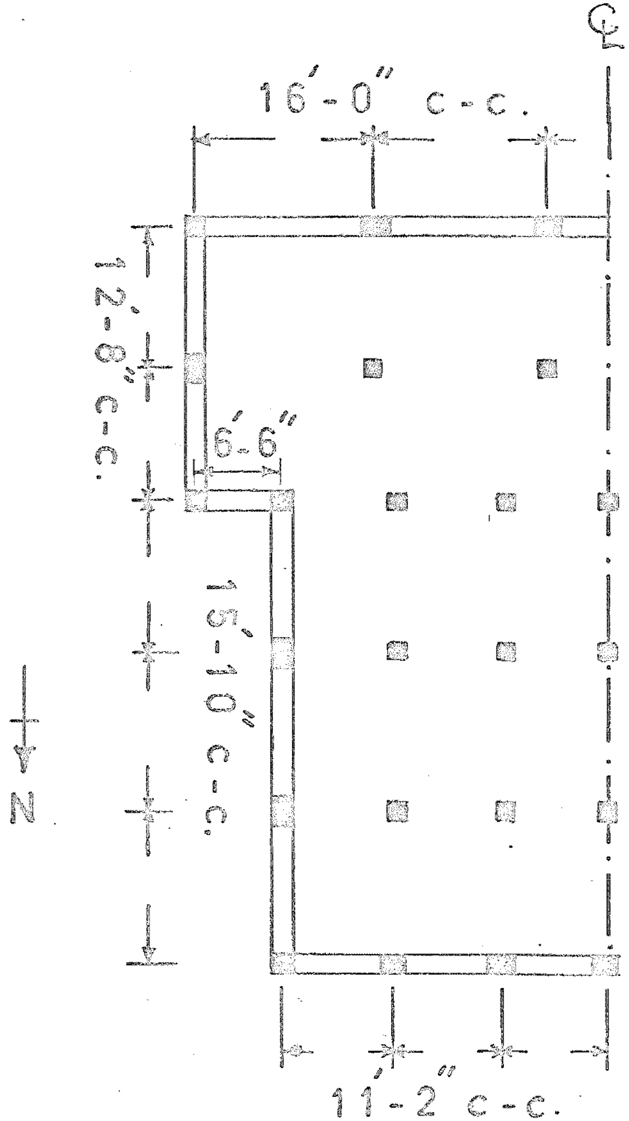


Figure 6·4

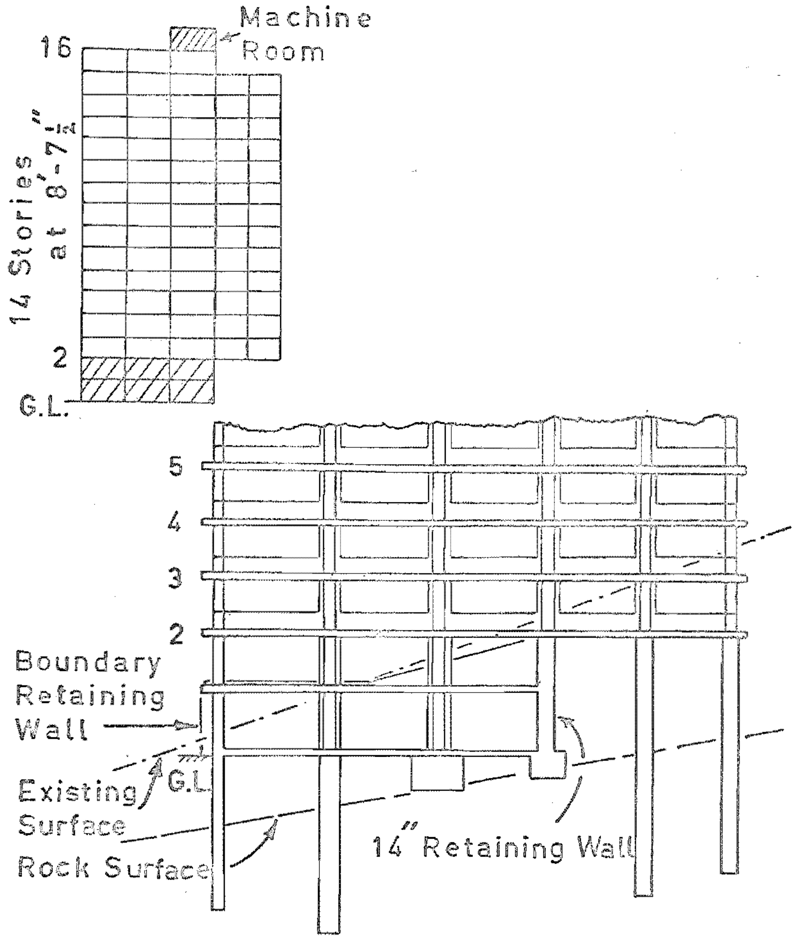
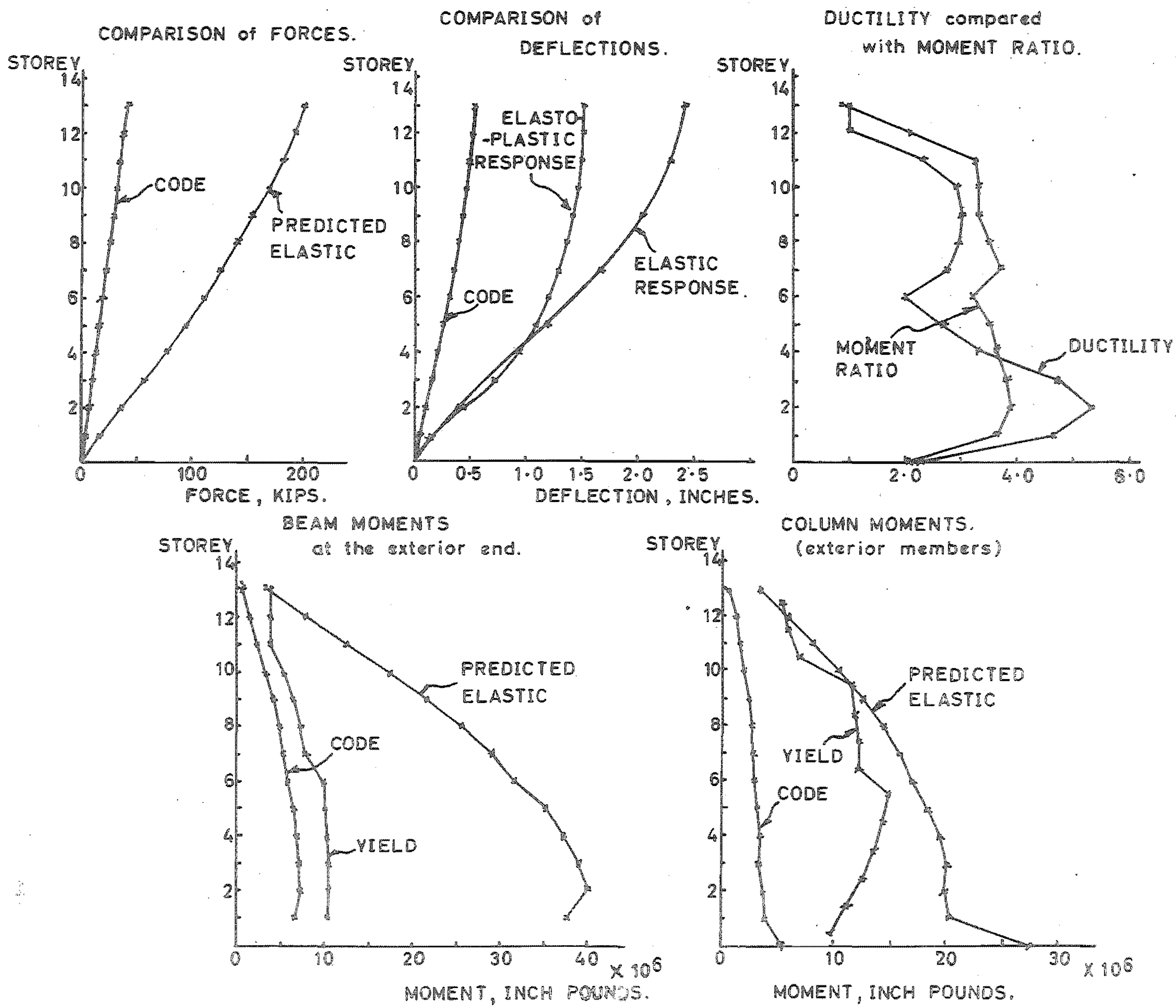


FIGURE 6·5


COMPARISON of FORCES.
STOREY
CODE
PREDICTED ELASTIC
FORCE, KIPS.
COMPARISON of DEFLECTIONS.
STOREY
ELASTO-PLASTIC RESPONSE
CODE
ELASTIC RESPONSE.
DEFLECTION, INCHES.
DUCTILITY compared with MOMENT RATIO.
STOREY
MOMENT RATIO
DUCTILITY
BEAM MOMENTS at the exterior end.
STOREY
PREDICTED ELASTIC
CODE
YIELD
X 10^6
MOMENT, INCH POUNDS.
COLUMN MOMENTS. (exterior members)
STOREY
PREDICTED ELASTIC
YIELD
CODE
X 10^6
MOMENT, INCH POUNDS.

DETAILS OF BEAM REINFORCEMENT BUILDING C ALL BARS HY60

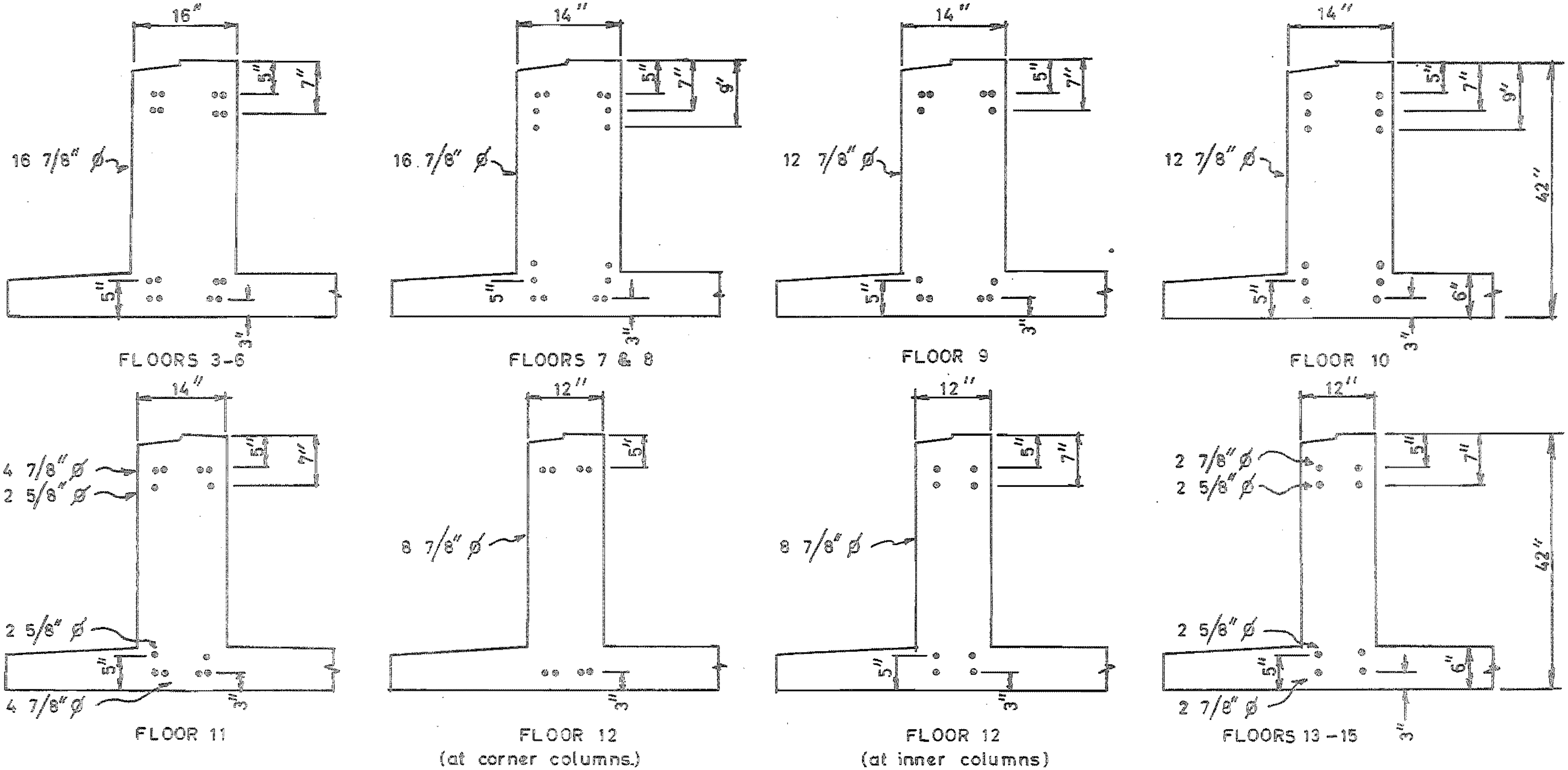


FIGURE 6-7

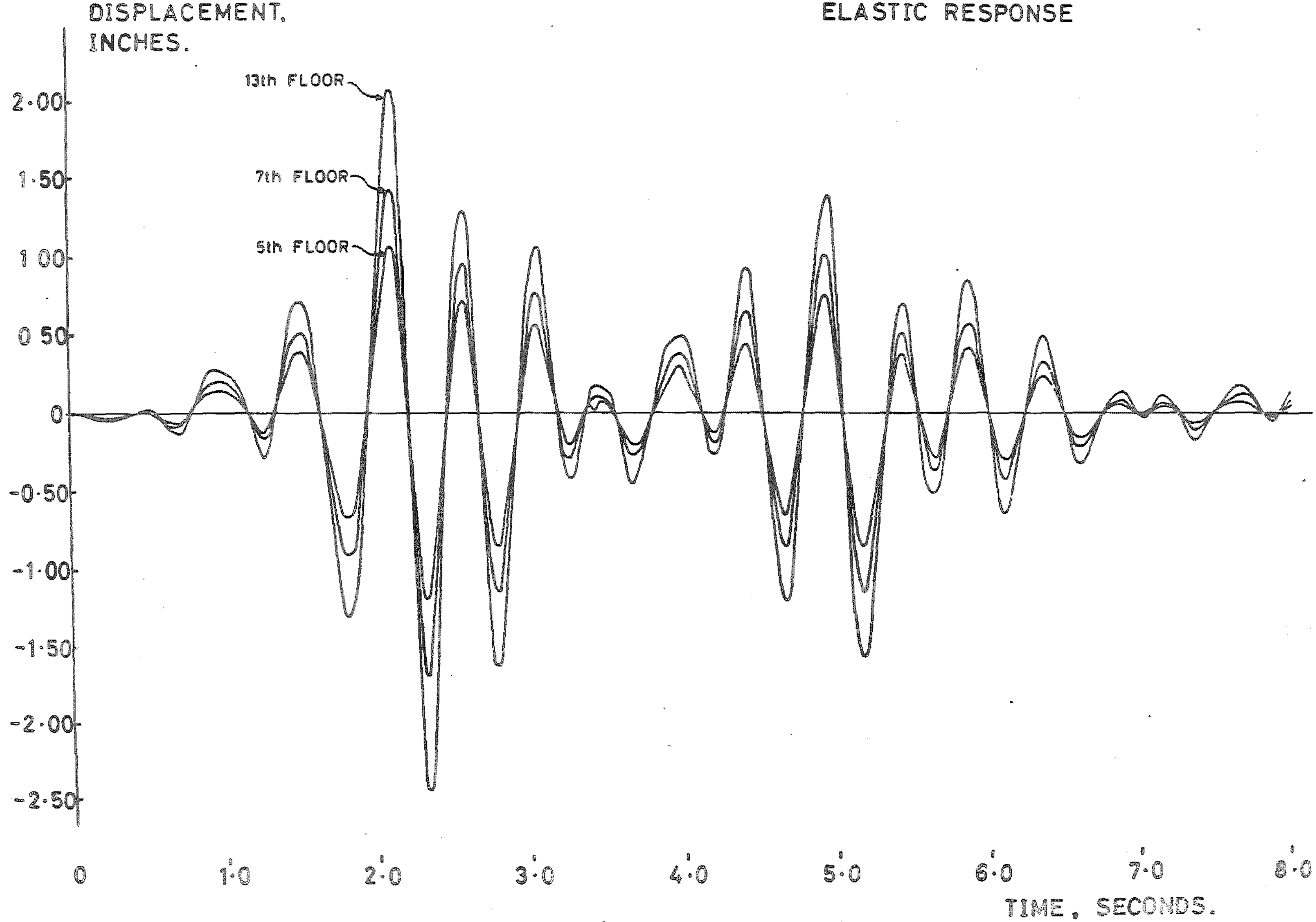

DISPLACEMENT, INCHES.
ELASTIC RESPONSE
13th FLOOR
7th FLOOR
5th FLOOR
2·00
1·50
1 00
0 50
0
-0·50
-1·00
-1·50
-2·00
-2·50
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
TIME, SECONDS.

FIGURE 6·8

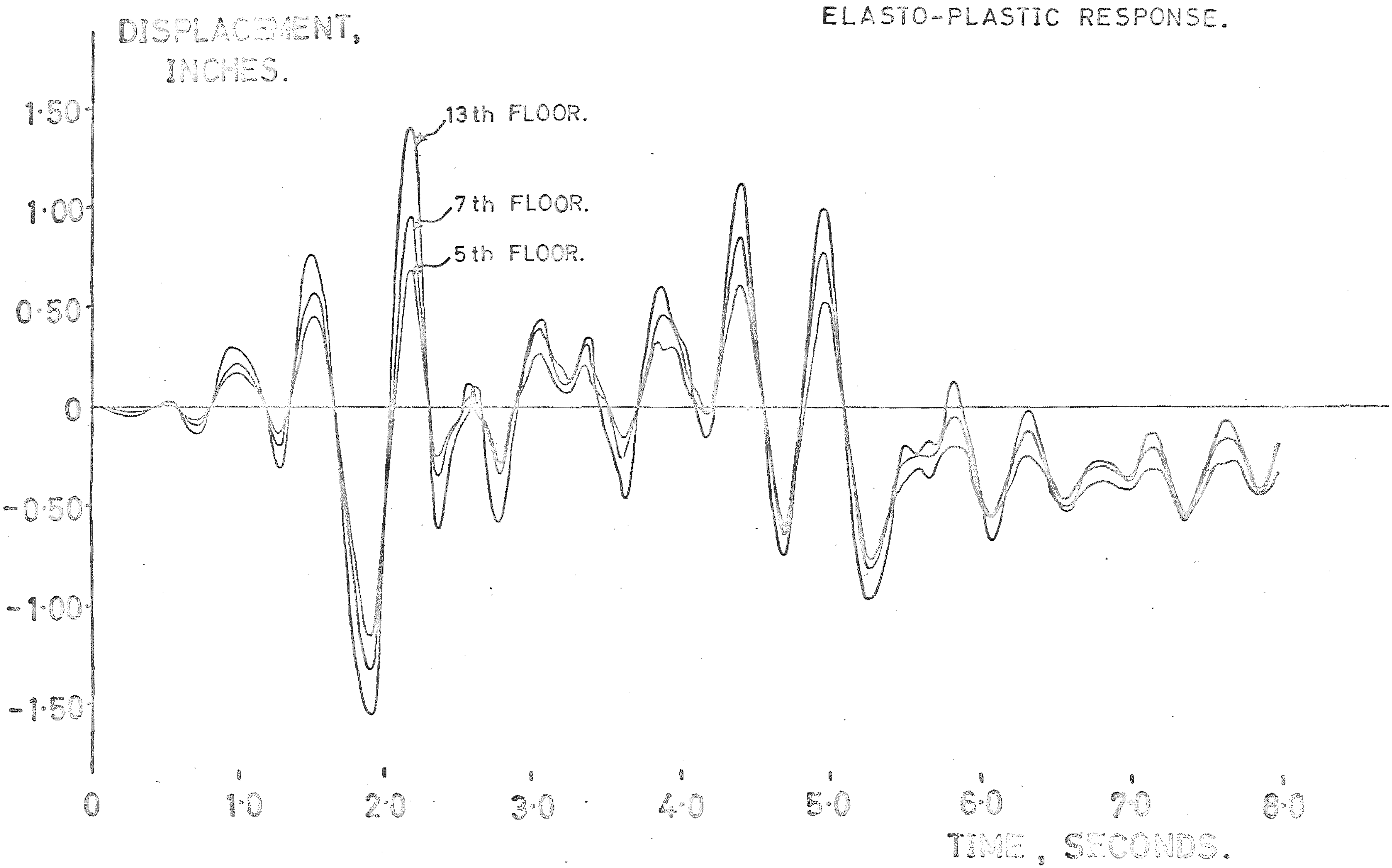
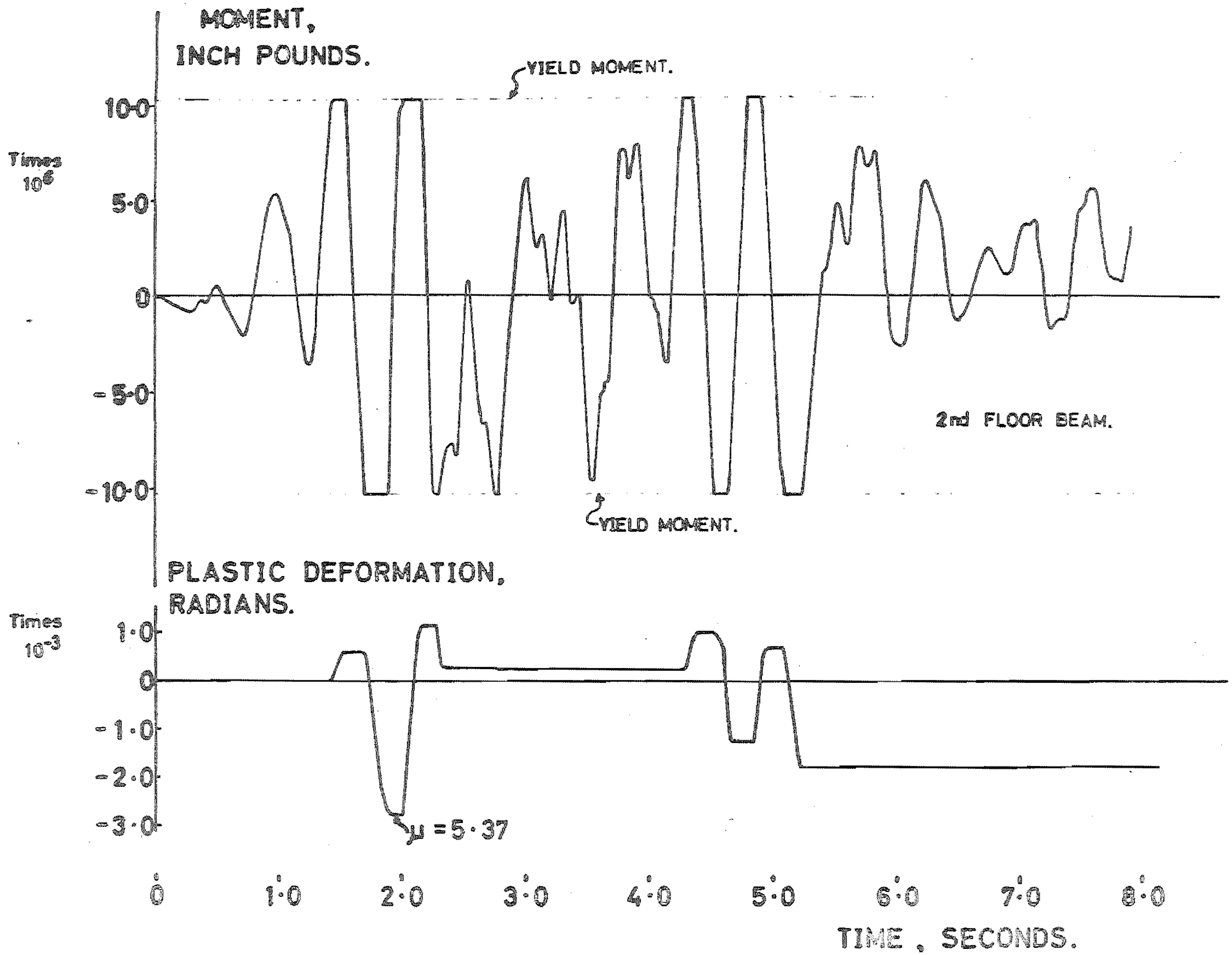

ELASTO-PLASTIC RESPONSE.
DISPLACEMENT, INCHES.
1·50
1·00
0·50
0
-0·50
-1·00
-1·50
13 th FLOOR.
7 th FLOOR.
5 th FLOOR.
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
TIME, SECONDS.

FIGURE 6·9



MOMENT,
INCH POUNDS.
YIELD MOMENT.
Times 10^6
10·0
5·0
0
-5·0
-10·0
YIELD MOMENT.
2nd FLOOR BEAM.
PLASTIC DEFORMATION,
RADIANS.
Times 10^{-3}
1·0
0
-1·0
-2·0
-3·0
$\mu = 5 \cdot 37$
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
TIME, SECONDS.

FIGURE 6·10

6.6 ANALYSIS OF BUILDING D

6.6.1 The Structure

This section considers the dynamic analysis of a transverse frame of a 12 storey reinforced concrete open frame building designed to be constructed in Auckland, New Zealand. The building is symmetrical in plan, with six bays in the longitudinal direction and two in the transverse, each floor having an area of 3600 sq.ft. The properties of the frame are given in Table 6.3. An outline of the elastic and elasto-plastic behaviour of this frame under the 1940 El Centro earthquake, N-S component, follows.

TABLE 6.3

Member Sizes

Columns		Beams	
12th Floor to 11th	14" x 14" I = 3200 in^4	12th Floor	12" x 24" I = 13,824 in^4
11th Floor to 10th	18" x 18" I = 8750 in^4	11th Floor to 9th	12" x 30" I = 27,000 in^4
10th Floor to 8th	20" x 20" I = 13,333 in^4	8th Floor to 6th	12" x 36" I = 46,656 in^4
8th Floor to 4th	24" x 24" I = 27,648 in^4	5th Floor to 3rd	15" x 36" I = 58,320 in^4
4th Floor to Ground	27" x 27" I = 44,280 in^4	2nd Floor to 1st	18" x 36" I = 69,984 in^4

TABLE 6.3 continued

Floor Mass per frame		Column Length inches	
Floor	Mass Kips	Floor	Length
12	32.2	12	
			11'0"
11	76.4	11	
			11'10"
10	79.8	10	
			11'10"
9	86.7	9	
			11'10"
8	90.5	8	
			11'10"
7	95.6	7	
			11'10"
6	95.6	6	
			11'10"
5	103	5	
			11'10"
4	103	4	
			11'10"
3	108	3	
			11'10"
2	108	2	
			13'7"
1	115	1	

All beams 22'4" long.

6.6.2 Elastic Analysis

A normal mode analysis was carried out to enable the seismic forces to be predicted from a response spectrum[24]. The member stiffnesses were estimated from the gross concrete sections takin the modulus of elasticity as 4.0×10^6 lb/in^2; the lateral flexibility matrix being calculated by the program "KLAT" which considered bending and shear deformations and the effect of rigid joints of finite size. The seismic forces were predicted by taking the root mean square of the modal responses found using the response spectrum curve assuming 10% critical damping. The member actions caused by these predicted forces were found by the computer program "MODTDE" and are given in Figure 6.10 together with the lateral forces. The normal mode properties and predicted response are given in Table 6.4.

The elastic response to the El Centro record was found by integrating the modal equations of motion and summing the responses at each increment using the program "SUMMOD" assuming 10% damping. The variation of the displacement of the floors with time is shown in Figure 6.11 and the maximum floor displacements are plotted in Figure 6.10. A static analysis was also carried out to determine the member actions under the lateral loading required by the N.Z. code NZSS 1900 Chapter 8[5]. Lateral loads were found using a coefficient of .065 on the base shear, as required for a private building of fundamental period 0.74 secs constructed in zone C, and were distributed as required by the code, i.e. in approximately triangular fashion.

The lateral loads, deflections and member actions derived from this code are compared with those predicted from the dynamic elastic response in Figure 6.10. A factor of 5.7 is needed to reduce the predicted elastic response to the code figures comparing base shear values.

6.6.3 Elasto-Plastic Analysis

The elasto-plastic response of the 12 storey 2 bay frame to the El Centro record was found with the computer program "JOINT" assuming 10% of critical damping in the first two modes. The ultimate moments of the members were assigned using a load factor of 1.25 on the NZSS code lateral, dead and seismic live loads for the beams and a load factor of 1.50 for the columns. The relationship of the yield moments to the predicted elastic seismic moments and the moments under code lateral load is illustrated in Figure 6.10.

The variation of the displacement of the floors is plotted against time in Figure 6.12. The maximum displacements assuming elastic action are compared with those assuming elasto-plastic action in Figure 6.10. The plastic action reduces the top storey deflection slightly, but the maximum displacement of the lower half of the frame is significantly increased. The displacement curves show a similar behaviour to the other reinforced concrete frames considered. The plastic action increases the magnitude of the first major peak at 1.90 secs, when the first yielding occurs, but reduces the magnitude of the following peaks. The response of this building differs from the others in that a major peak occurs at 5.40 secs when a large part of the plastic deformation in various members occurs. There is also a slight increase in the periodicity of motion.

The variation of the bending moment in a beam and column member is shown in Figure 6.13 with the growth of plastic deformation in each member plotted under the bending moment curve. After the increase in plastic deformations which occurs at 5.40 secs, a significant amount of permanent set is evident, totalling approximately 1.5" at the top floor.

The major part of the yielding occurs in the beam members with a maximum ductility factor of 7.01 in the first floor beam. A large ductility was also required at the base of the bottom column with a maximum of 9.48. The next largest column ductility, of 2.01, occurs just above the second floor.

It would appear for this frame that except for the bottom columns the plastic deformation has been largely restricted to the beams by making these members relatively weaker than the columns. The ductility ratio of the beam members is compared to the ratio of the maximum moment assuming elastic response to the yield moment in Figure 6.10. It can be seen that there is a pronounced tendency for the ductility requirements to be largest in the members with the highest moment ratio and for the members with a high moment ratio to require a ductility greater than the moment ratio.

TABLE 6.4

NORMAL MODE PROPERTIES

Mode 1

Frequency = 1.35 c.p.s. Period = 0.739 secs.

Amplification Factor = 0.47

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	7.791	47.0
.966	7.530	154.7
.905	7.054	260.2
.820	6.931	363.8
.724	5.640	459.6
.644	5.014	549.2
.551	4.289	626.2
.452	3.519	694.0
.351	2.733	746.8
.257	2.002	787.2
.165	1.282	813.0
.076	.589	825.6

Mode 2

Frequency = 3.38 c.p.s. Period = 2.96 secs.

Amplification Factor = 0.72

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	-.635	-23.9
.800	-.508	-69.2
.480	-.305	-97.6
.107	-.068	-104.5
-.227	.144	-89.2
-.411	.261	-60.1
-.532	.338	-22.4
-.577	.367	21.7
-.540	.343	62.9
-.445	.283	98.4
-.309	.196	123.0
-.148	.094	135.6

TABLE 6.4 continued

Mode 3

Frequency = 5.62 c.p.s. Period = 0.178 secs.

Amplification Factor = 0.64

Displacement Ratios	1g Displacements inches	1g Shears Kips
1.000	.1536	15.96
.489	.0751	34.46
-.113	-.0174	30.00
-.548	-.0841	6.49
-.650	-.0998	-22.68
-.466	-.0716	-44.76
-.118	-.0181	-50.34
.251	.0386	-37.52
.504	.0774	-11.78
.571	.0876	18.65
.474	.0729	43.96
.250	.0384	58.22

PREDICTED ELASTIC RESPONSE

Displacement inches	Shears Kips	Forces Kips
3.743	30.0	30.04
3.607	91.8	61.70
3.369	143.9	52.12
3.047	189.1	45.22
2.691	228.6	39.64
2.397	266.8	38.18
2.058	300.6	33.66
1.698	332.0	31.42
1.327	358.8	26.84
.978	382.0	23.10
.629	398.4	16.51
.290	407.2	8.72

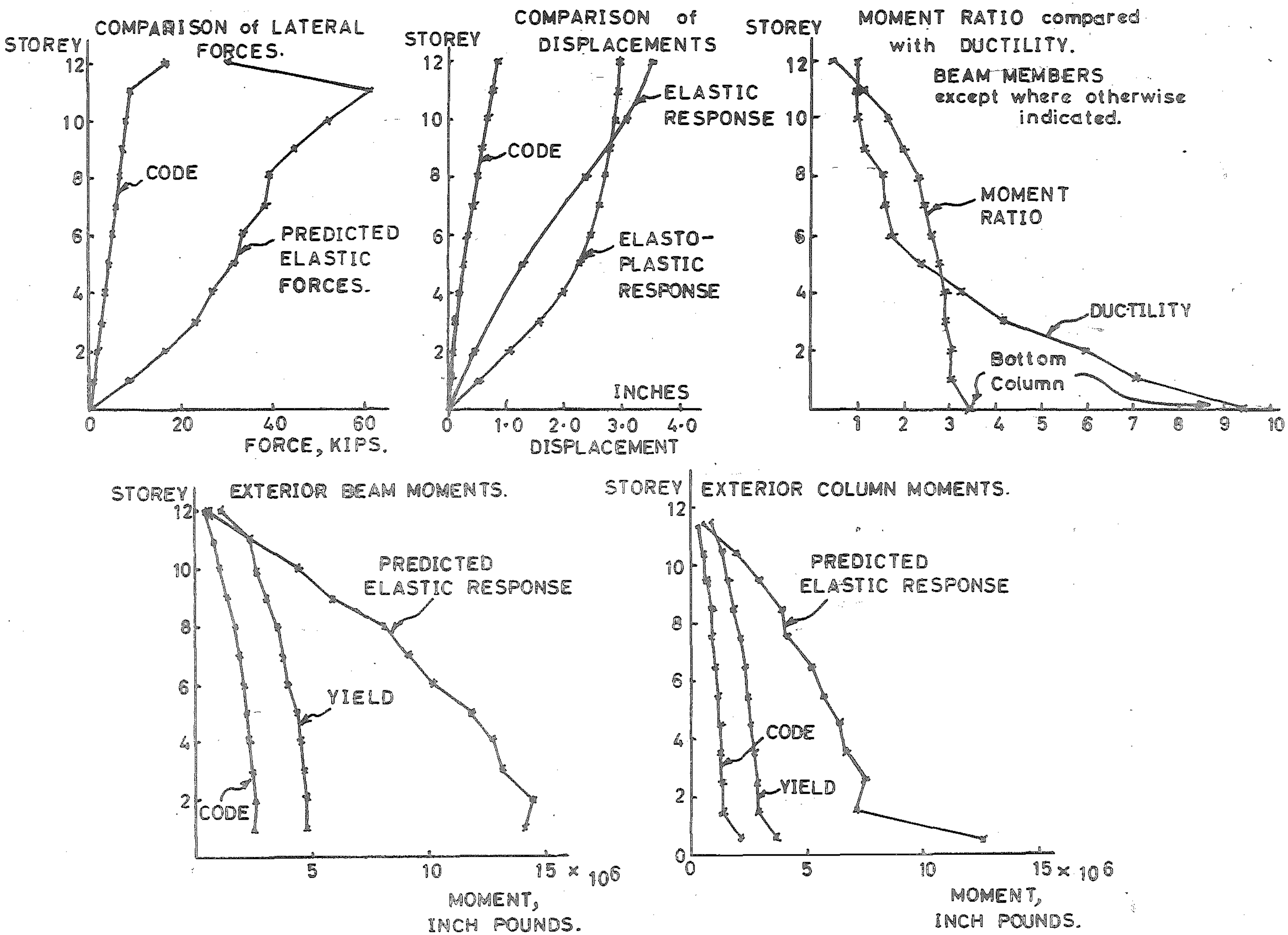
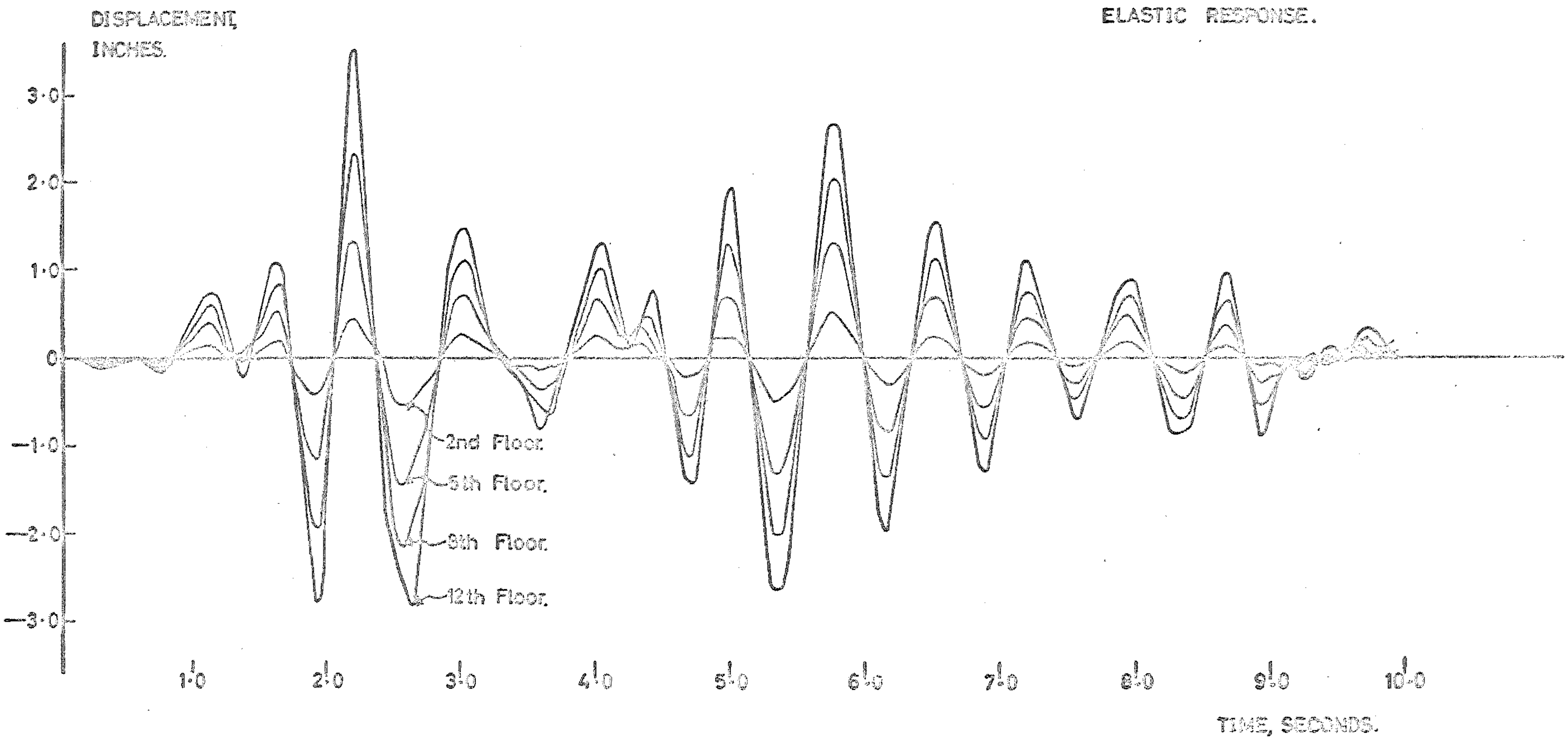

COMPARISON of LATERAL FORCES.
STOREY
CODE
PREDICTED ELASTIC FORCES.
FORCE, KIPS.
COMPARISON of DISPLACEMENTS
ELASTIC RESPONSE
CODE
ELASTO-PLASTIC RESPONSE
INCHES
DISPLACEMENT
MOMENT RATIO compared with DUCTILITY.
BEAM MEMBERS except where otherwise indicated.
MOMENT RATIO
DUCTILITY
Bottom Column
EXTERIOR BEAM MOMENTS.
PREDICTED ELASTIC RESPONSE
YIELD
CODE
MOMENT, INCH POUNDS.
EXTERIOR COLUMN MOMENTS.
PREDICTED ELASTIC RESPONSE
CODE
YIELD
MOMENT, INCH POUNDS.

FIGURE 6·11



DISPLACEMENT, INCHES.
ELASTIC RESPONSE.
3·0
2·0
1·0
0
−1·0
−2·0
−3·0
2nd Floor.
5th Floor.
9th Floor.
12th Floor.
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 6·12

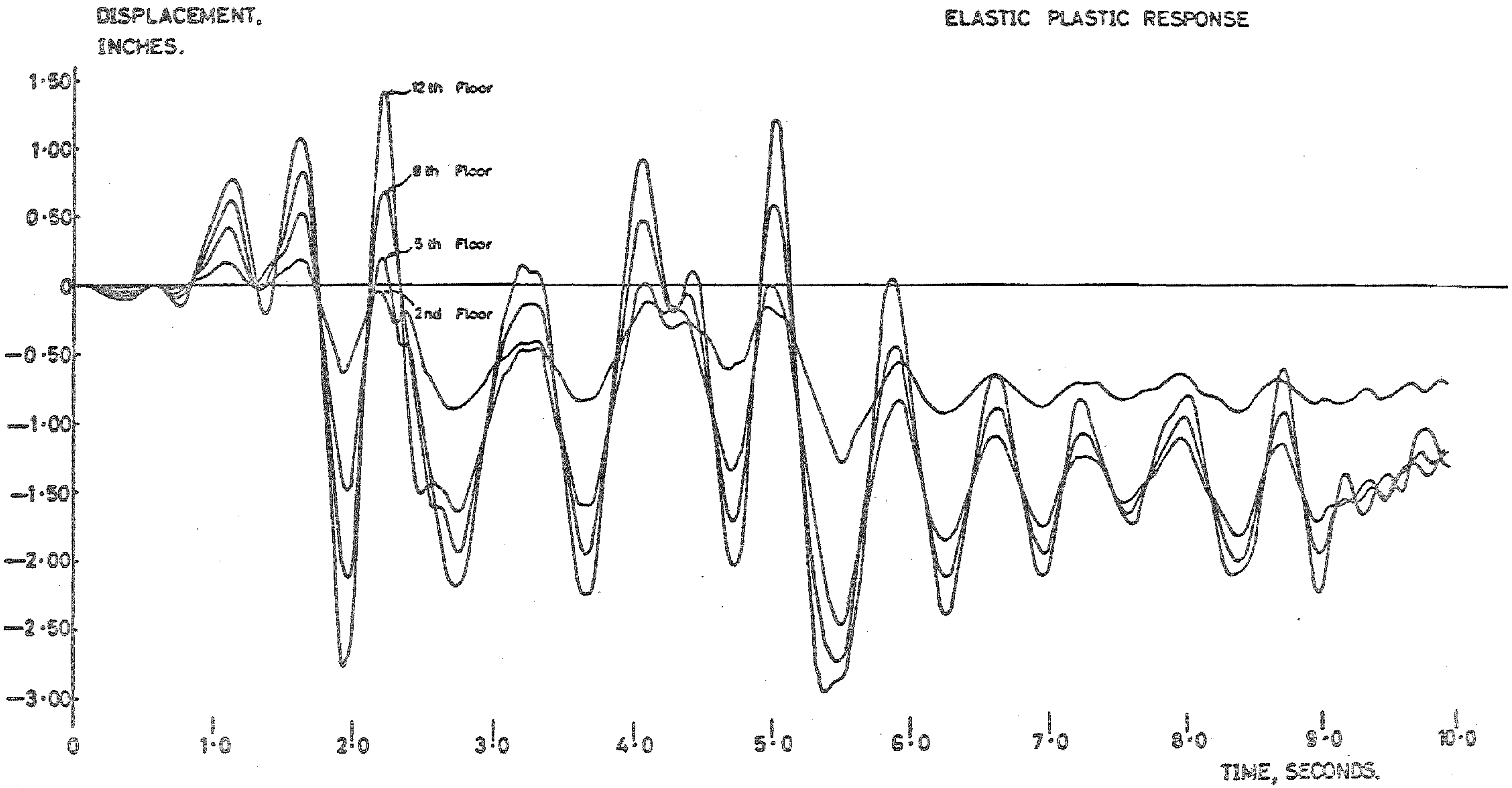
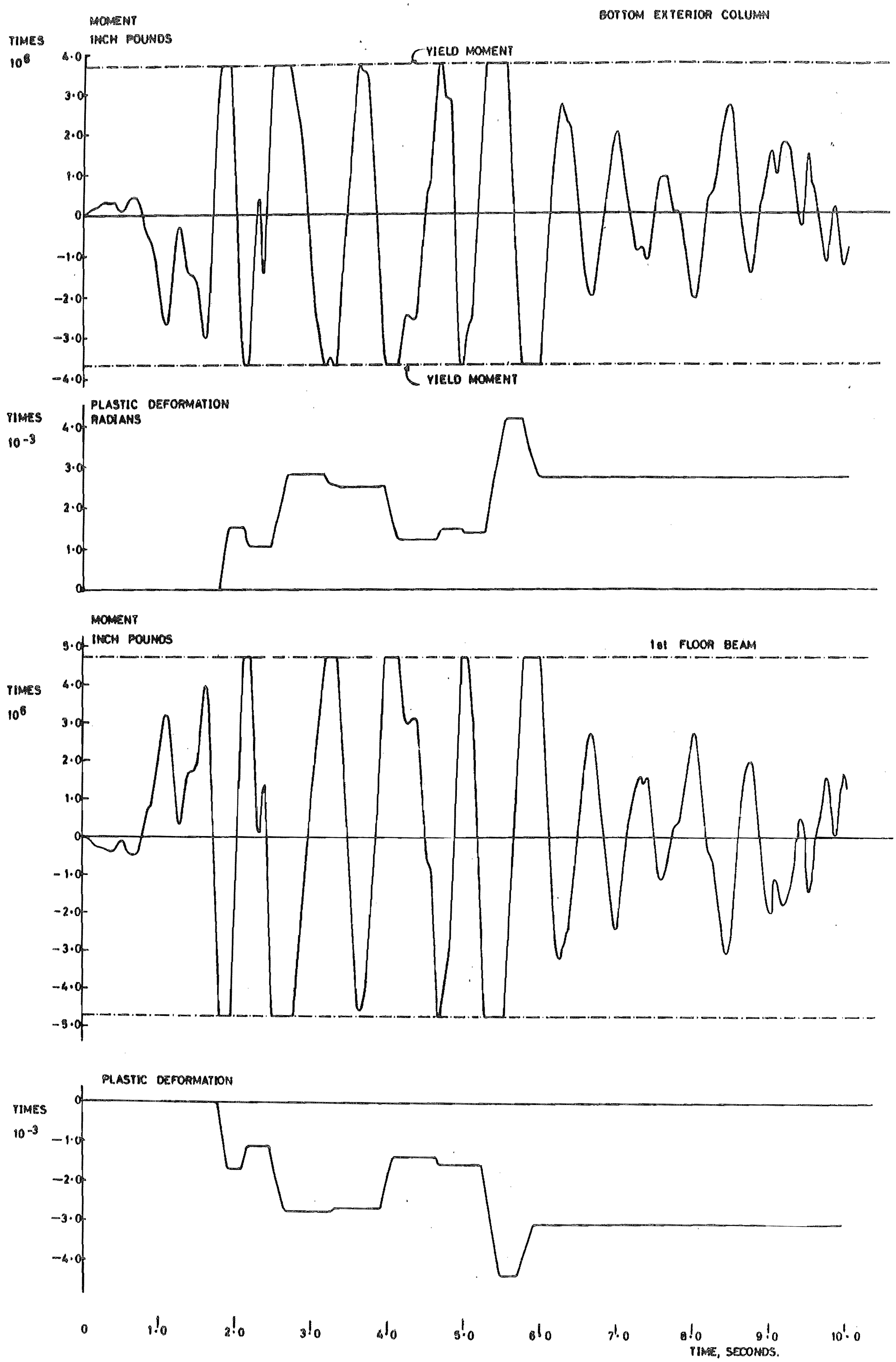

DISPLACEMENT,
INCHES.
ELASTIC PLASTIC RESPONSE
12 th Floor
8 th Floor
5 th Floor
2nd Floor
1·50
1·00
0·50
0
−0·50
−1·00
−1·50
−2·00
−2·50
−3·00
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 6·13



BOTTOM EXTERIOR COLUMN
MOMENT
INCH POUNDS
TIMES
10^6
YIELD MOMENT
YIELD MOMENT
4·0
3·0
2·0
1·0
0
-1·0
-2·0
-3·0
-4·0
PLASTIC DEFORMATION
RADIANS
TIMES
10^{-3}
MOMENT
INCH POUNDS
1st FLOOR BEAM
5·0
-5·0
PLASTIC DEFORMATION
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.

FIGURE 6·14

LIST 11

```
C      PROGRAM ''JOINT''    LIST 11.                        PAGE 1
C      LANGUAGE - FORTRAN IV FOR IBM 360/44
C      THIS PROGRAM CALCULATES THE ELASTO/PLASTIC RESPONSE OF A
C      REGULAR MULTI-STOREY FRAME TO AN EARTHQUAKE RECORD.
C
       DIMENSION T(5),GR(5),TSD(18),SD(19),VEL(6),ACC(6),S(18,18),
      1C(6,6),AA(6),BB(6),W(6),DR(18),IP1(18),IP2(18),PDM1(18),
      2PDM2(18),PD1(18),PD2(18),IS(18),JS(18),KS(18),LS(18),P1(18),
      3AS(18),Q1(18),H1(18),YM1(18),YM2(18),IH1(18),IH2(18),R1(18),R2(18)
       DOUBLE PRECISION SR,SE,DT,TR,E,TM,XMAX,PP,DG2,DG,TT,TS,GS,GG,P,Q,
      1H,BETA,R,G,F,T2,T,GR,TSD,SD,VEL,ACC,S,C,AA,BB,W,DR,PDM1,PDM2,PD1,
      2PD2,P1,AS,Q1,H1,YM1,YM2,R1,R2
       INP=5
       LNP=6
       IPCH=7
C      N=NO OF DEFORMATIONS,NS=NO OF STOREYS,NB=NO OF BAYS
C      M4=M3=NO OF MEMBERS,KP=NO OF STEPS BEFORE PRINTING,
C      SR=G/E,SE=EFFECTIVE SHEAR AREA FACTOR,DT=STEP INTERVAL,
C      TR=LENGTH OF EQU. RECORD,E=YOUNGS MODULUS
   731 READ(INP,30) N,NS,NB,M4,M3,KP,SR,SE,DT,TR,E
    30 FORMAT(6I3,F4.2,F4.1,3E10.4)
       READ(INP,500) I1,I2,I3,I4,I5,I6,I7
       READ(INP,500) J1,J2,J3,J4,J5,J6,J7
   500 FORMAT(7I3)
       SD(N&1)=0.
       KC=KP
       NB2=NB&2
       NB1=NB&1
       ISQ=1
       DO171M=1,M3
       PD1(M)=0.
   171 PD2(M)=0.
       WRITE(LNP,330) N,NS,NB,M4,M3,KP,SR,SE,DT,TR,E
   330 FORMAT(1X,6I4,F5.2,F5.1,3E13.4///)
       WRITE(LNP,501) I1,I2,I3,I4,I5,I6,I7
       WRITE(LNP,501) J1,J2,J3,J4,J5,J6,J7
   501 FORMAT(1X,7I4)
C      READ AND STORE MEMBER DATA
       DO28M=1,M4
C      IS,JS,KS,LS=FRAME DEFORMATIONS ASSOCIATED WITH MEMBER,
C      P1=MOMENT OF INERTIA,H1=CENTRE LINE LENGTH,Q1=CLEAR LENGTH,
C      YM1=YM2 MEMBER YIELD MOMENT REDUCED BY 1000,IH1,IH2=1 OR 0
C      FOR HINGES OR NOT AT ENDS,R1,R2 INITIAL B.M. AT ENDS.
       READ(INP,31) IS(M),JS(M),KS(M),LS(M),P1(M),H1(M),YM1(M),YM2(M),
      1IH1(M),IH2(M),R1(M),R2(M),AS(M),Q1(M)
    31 FORMAT(4I3,4E6.0,2I2,4E10.0)
       WRITE(LNP,331) IS(M),JS(M),KS(M),LS(M),P1(M),AS(M),H1(M),Q1(M),
      1YM1(M),YM2(M),IH1(M),IH2(M),R1(M),R2(M)
   331 FORMAT(1X,4I4,6E13.4,2I3,2E13.4)
       YM1(M)=YM1(M)*1.E3
    28 YM2(M)=YM2(M)*1.E3
       WRITE(LNP,341)
       DO29I=1,NS
C      READ DAMPING MATRIX
       READ(INP,32) (C(I,J),J=1,NS)
       WRITE(LNP,332) (C(I,J),J=1,NS)
    29 WRITE(LNP,340)
    32 FORMAT(5E14.8)
   332 FORMAT(1X,5E18.8)
   340 FORMAT(2X)
       WRITE(LNP,341)
   341 FORMAT(//)
C      READ MASS MATRIX
```

```
C      PROGRAM ''JOINT''    LIST 11.                        PAGE 2
       READ(INP,33) (W(I),I=1,NS)
    33 FORMAT(8E10.4)
       WRITE(LNP,333) (W(I),I=1,NS)
   333 FORMAT(1X,8E13.4/)
       WRITE(LNP,341)
       DO40I=1,NS
    40 W(I)=W(I)/386.4
       DO41I=1,M3
       PDM1(I)=0.
       PDM2(I)=0.
       IP1(I)=0
    41 IP2(I)=0
       DO42I=1,N
    42 TSD(I)=0.
       DO43I=1,NS
    43 VEL(I)=0.
       TM=0.
       XMAX=0.
       MSK=1
       ICC=1
C      READ EQU. RECORD CARD
       READ(INP,35) ISC,(T(I),GR(I),I=2,5)
    35 FORMAT(I3,4(F8.4,F9.6))
       IF(ICC-ISC)44,45,44
    45 M2=2
       PP=GR(M2)*386.4
       DG2=(GR(3)-GR(2))/(T(3)-T(2))*DT*386.4
       DO46I=1,NS
    46 ACC(I)=-GR(2)*386.4
    47 KC=KC&1
       IF(KC-KP)356,357,357
   357 KC=0
       WRITE(LNP,340)
       WRITE(LNP,334) TM,(TSD(I),I=NB2,N,NB2)
   334 FORMAT(1X,8E13.4)
       IF(I7-1)502,503,504
   504 WRITE(IPCH,343) TM,TSD(I1),TSD(I2),TSD(I3),TSD(I4),
      1TSD(I5),TSD(I6),ISQ
   343 FORMAT(7E10.3,5X,I5)
       WRITE(IPCH,345) R1(J1),R1(J2),R1(J3),R1(J4),
      1R2(J5),R1(J6),R2(J7),ISQ
   345 FORMAT(7E10.3,I5)
   503 WRITE(LNP,346) R1(J1),R1(J2),R1(J3),R1(J4),
      1R2(J5),R1(J6),R2(J7),ISQ
   346 FORMAT(1X,7E13.4,25X,I5)
       WRITE(LNP,334) PD1(J1),PD1(J2),PD1(J3),PD1(J4),
      1PD2(J5),PD1(J6),PD2(J7)
   502 ISQ=ISQ&1
   356 IF(TM&DT-T(M2&1))48,48,49
    48 TM=TM&DT
       DG=DG2
       PP=PP&DG
       GO TO 50
    49 IF(M2-4)80,81,81
    80 M2=M2&1
       GO TO 83
    81 M2=1
       GR(1)=GR(5)
       T(1)=T(5)
       ICC=ICC&1
C      READ EQU. RECORD CARD
       READ(INP,35) ISC,(T(I),GR(I),I=2,5)
```

```
      IF(ICC-ISC)44,83,44
   44 WRITE(LNP,337) ICC
  337 FORMAT(1X,19HCARDS OUT OF ORDER.I5)
      STOP
   83 TT=TM&DT-T(M2)
      TS=T(M2&1)-T(M2)
      IF(TS)49,49,84
   84 GS=(GR(M2&1)-GR(M2))*386.4
      GG=GS*TT/TS
      DG2=GS*DT/TS
      TM=TM&DT
      DG=GR(M2)*386.4&GG-PP
      PP=GR(M2)*386.4&GG
   50 DO85I=1,NS
      AA(I)=-6.0*VEL(I)/DT-3.0*ACC(I)
   85 BB(I)=-3.0*VEL(I)-0.5*DT*ACC(I)
      DO86I=1,NS
      DO82K=1,NB1
      L=(I-1)*NB2&K
   82 DR(L)=0.
      GS=0.
      DO87J=1,NS
   87 GS=GS&C(I,J)*BB(J)
      K=I*NB2
   86 DR(K)=-W(I)*DG-W(I)*AA(I)-GS
      GO TO (88,89),MSK
C     SET UP FRAME STIFFNESS MATRIX
   88 WRITE(LNP,340)
      WRITE(LNP,333) TM
      WRITE(LNP,106) (IH1(I),IH2(I),I=1,M3)
  106 FORMAT(1X,36I3)
      DO140I=1,N
      DO140J=1,N
  140 S(I,J)=0.
      DO27M=1,M4
      I=IS(M)
      J=JS(M)
      K=KS(M)
      L=LS(M)
      P=P1(M)
      Q=Q1(M)
      H=H1(M)
      BETA=6.0*P*SE/(Q*Q*AS(M)*SR)
      IF(IH1(M)-1)3,4,4
    3 IF(IH2(M)-1)5,8,8
    4 IF(IH2(M)-1)6,27,27
    5 P=2.0*P*E/(Q*(1.0&2.0*BETA))
      R=P*(2.0&BETA)
      P=P*(1.0-BETA)
      G=(R&P)/(Q*Q)
      F=0.5*(H&Q)*(H-Q)
      S(K,K)=S(K,K)&R&G*F
      IF(L-N)7,7,9
    7 S(K,L)=S(K,L)-G*H
      S(L,K)=S(K,L)
      S(L,L)=S(L,L)&2.0*G
      IF(J-N)10,10,27
   10 S(J,K)=S(J,K)&G*H
      S(K,J)=S(J,K)
      S(J,J)=S(J,J)&2.0*G
      S(J,L)=S(J,L)-2.0*G
      S(L,J)=S(J,L)
```

LIST 11


```
      S(I,J)=S(I,J)&G*H
      S(J,I)=S(I,J)
      S(I,L)=S(I,L)-G*H
      S(L,I)=S(I,L)
    9 S(I,I)=S(I,I)&R&G*F
      S(I,K)=S(I,K)&P&G*F
      S(K,I)=S(I,K)
      GO TO 27
    6 P=3.0*P*E/(Q**3*(1&0.5*BETA))
      G=H-Q
      F=H&Q
      S(K,K)=S(K,K)&0.25*P*F*F
      IF(L-N)11,11,12
   11 S(K,L)=S(K,L)-0.5*P*F
      S(L,K)=S(K,L)
      S(L,L)=S(L,L)&P
      IF(J-N)13,13,27
   13 S(J,K)=S(J,K)&0.5*P*F
      S(K,J)=S(J,K)
      S(J,J)=S(J,J)&P
      S(J,L)=S(J,L)-P
      S(L,J)=S(J,L)
      S(I,J)=S(I,J)&0.5*P*G
      S(J,I)=S(I,J)
      S(I,L)=S(I,L)-0.5*P*G
      S(L,I)=S(I,L)
   12 S(I,I)=S(I,I)&0.25*P*G*G
      S(I,K)=S(I,K)&0.25*P*G*F
      S(K,I)=S(I,K)
      GO TO 27
    8 P=3.0*P*E/(Q**3*(1.0&0.5*BETA))
      G=H-Q
      F=H&Q
      S(K,K)=S(K,K)&0.25*P*G*G
      IF(L-N)15,15,16
   15 S(K,L)=S(K,L)-0.5*P*G
      S(L,K)=S(K,L)
      S(L,L)=S(L,L)&P
      IF(J-N)14,14,27
   14 S(J,K)=S(J,K)&0.5*P*G
      S(K,J)=S(J,K)
      S(J,J)=S(J,J)&P
      S(J,L)=S(J,L)-P
      S(L,J)=S(J,L)
      S(I,J)=S(I,J)&0.5*P*F
      S(J,I)=S(I,J)
      S(I,L)=S(I,L)-0.5*P*F
      S(L,I)=S(I,L)
   16 S(I,I)=S(I,I)&0.25*F*F*P
      S(I,K)=S(I,K)&0.25*P*G*F
      S(K,I)=S(I,K)
   27 CONTINUE
      DO90I=1,NS
      K=I*NB2
      S(K,K)=S(K,K)&6.0*W(I)/(DT*DT)
      DO90J=1,NS
      L=J*NB2
   90 S(K,L)=S(K,L)&3.0*C(I,J)/DT
C     INVERTS (S) IN SITU. ORDER NXN.
      DO101I=1,N
      T2=S(I,I)
      IF(T2)350,351,350
```

```
351 WRITE(LNP,339)
339 FORMAT(1X,15HSINGULAR MATRIX////)
    STOP
350 S(I,I)=1.0
    DO102J=1,N
102 S(I,J)=S(I,J)/T2
    DO101L=1,N
    IF(L-I)103,101,103
103 T2=S(L,I)
    S(L,I)=0.
    DO104J=1,N
104 S(L,J)=S(L,J)-T2*S(I,J)
101 CONTINUE
 89 DO91I=1,N
    GS=0.
    DO92J=1,N
 92 GS=GS&S(I,J)*DR(J)
    SD(I)=GS
 91 TSD(I)=TSD(I)&SD(I)
    DO93I=1,NS
    J=I*(NB&2)
    VEL(I)=VEL(I)&3.0*SD(J)/DT&BB(I)
 93 ACC(I)=ACC(I)&6.0*SD(J)/(DT*DT)&AA(I)
    IF(DABS(TSD(N)).LE.DABS(XMAX)) GO TO 112
113 XMAX=TSD(N)
    TMAX=TM
112 DO78M=1,M3
    I=IS(M)
    J=JS(M)
    K=KS(M)
    L=LS(M)
    P=P1(M)
    Q=Q1(M)
    BETA=6.0*P*SE/(Q*Q*SR*AS(M))
    H=H1(M)
    IF(IH1(M)-1)51,52,52
 51 IF(IH2(M)-1)53,54,54
 52 IF(IH2(M)-1)55,56,56
 54 P=3.0*P*E/(Q*Q*(1.0&0.5*BETA))
    G=H-Q
    F=H&Q
    R1(M)=R1(M)&0.5*P*F*SD(I)&0.5*P*G*SD(K)&P*(SD(J)-SD(L))
    Q=(1.0&1.5*G/(Q*(2.&BETA)))*SD(K)
   1&(1.-BETA&1.5*G/Q)*SD(I)/(2.&BETA)&3.*(SD(J)-SD(L))/((2.&BETA)*Q)
    MM=1
 58 IF(R2(M)*Q)60,62,62
 60 IH2(M)=0
    GO TO 66
 62 PD2(M)=PD2(M)&Q
    IF(DABS(PD2(M)).GT.DABS(PDM2(M))) PDM2(M)=PD2(M)
 66 GO TO (166,67),MM
166 IF(DABS(R1(M)).LE.YM1(M)) GO TO 63
 76 IH1(M)=1
    R1(M)=.9999999*YM1(M)*R1(M)/DABS(R1(M))
 63 GO TO 78
 55 P=3.0*P*E/(Q*Q*(1.0&0.5*BETA))
    G=H-Q
    F=H&Q
    R2(M)=R2(M)&0.5*P*G*SD(I)&0.5*P*F*SD(K)&P*(SD(J)-SD(L))
    Q=(1.0&1.5*G/(Q*(2.&BETA)))*SD(I)
   1&(1.-BETA&1.5*G/Q)*SD(K)/(2.&BETA)&3.*(SD(J)-SD(L))/((2.&BETA)*Q)
    MM=1
```



```
 68 IF(R1(M)*Q)72,74,74
 72 IH1(M)=0
    GO TO 79
 74 PD1(M)=PD1(M)&Q
    IF(DABS(PD1(M)).GT.DABS(PDM1(M))) PDM1(M)=PD1(M)
 79 GO TO (179,78),MM
179 IF(DABS(R2(M)).LE.YM2(M)) GO TO 78
170 IH2(M)=1
    R2(M)=.9999999*YM2(M)*R2(M)/DABS(R2(M))
    GO TO 78
 56 Q=0.5*(H-Q)*SD(I)/Q&(1.0&0.5*(H-Q)/Q)*SD(K)&(SD(J)-SD(L))/Q
    MM=2
    GO TO 58
 67 Q=Q1(M)
    Q=(1.0&0.5*(H-Q)/Q)*SD(I)&0.5*(H-Q)*SD(K)/Q&(SD(J)-SD(L))/Q
    GO TO 68
 53 P=2.0*P*E/(Q*(1.0&2.0*BETA))
    R=P*(2.0&BETA)
    P=P*(1.0-BETA)
    G=(R&P)/Q
    F=(H-Q)*0.5
    R1(M)=R1(M)&(R&G*F)*SD(I)&(P&G*F)*SD(K)&G*(SD(J)-SD(L))
    R2(M)=R2(M)&(P&G*F)*SD(I)&(R&G*F)*SD(K)&G*(SD(J)-SD(L))
    IF(DABS(R1(M)).LE.YM1(M)) GO TO 79
 77 IH1(M)=1
    R1(M)=.9999999*YM1(M)*R1(M)/DABS(R1(M))
    GO TO 79
 78 CONTINUE
    IF(TM-TR&.5*DT)94,95,95
 94 MSK=2
    DO100I=1,M3
    IF(IH1(I)-IP1(I))198,99,198
 99 IF(IH2(I)-IP2(I))198,199,198
198 MSK=1
199 IP1(I)=IH1(I)
100 IP2(I)=IH2(I)
    GO TO 47
 95 WRITE(LNP,344)
344 FORMAT(1H1)
C   PRINT MAX. DEFLECTION
    WRITE(LNP,333) XMAX,TMAX
    WRITE(LNP,341)
    DO96I=1,M3
    BETA=12.0*P1(I)*SE/(Q1(I)*SR*AS(I))&Q1(I)
    U1=1.0&DABS(PDM1(I))*P1(I)*6.0*E/(YM1(I)*BETA)
    U2=1.0&DABS(PDM2(I))*P1(I)*6.0*E/(YM2(I)*BETA)
C   PRINT MEMBER DUCTILITIES.
 96 WRITE(LNP,336) I,U1,U2
336 FORMAT(1X,I3,2F10.2)
    READ(INP,730) NFIN
730 FORMAT(I3)
    GO TO (733,732),NFIN
733 WRITE(LNP,344)
    GO TO 731
732 WRITE(LNP,338)
338 FORMAT(//1X,19HPROCESSING COMPLETE////)
    STOP
    END
```

LIST 11

CHAPTER SEVEN

DYNAMIC ANALYSIS OF A RAILWAY BRIDGE

7.1 INTRODUCTION

This chapter describes the dynamic analysis of a proposed design for a railway bridge for the New Zealand North Island Main Trunk. The structure consists of a continuous reinforced concrete deck beam of fourteen 40 feet spans, supported by single reinforced concrete cylindrical piers, except that three of the piers consist of two inclined cylindrical members, as shown in the elevations in Figure 7.1. The bridge was designed so that plastic hinges would form in the deck beam under a strong motion earthquake, acting transversely to the deck.

7.2 GENERAL ASSUMPTIONS

It was assumed that the axial deformation in all beams and vertical piers, and rotations in the plane of the structure were negligible because there was no loading corresponding to these deformations. Shear deformations and joint size effects were neglected for simplicity. The pad footings were assumed to provide a pin joint and the belled piles to rigidly fix the piers a little below ground level. Axial and bending deformations were considered in the inclined piers which were assumed to be rigidly fixed at the tie beams at ground level. Figure 7.4 shows the 16 lateral co-ordinates used to define the displaced shape of the structure. There were also two rotations associated with each lateral displacement, one about the vertical pier axis and one about the deck beam axis. This means that when the equations of equilibrium for the structure are written it is necessary to consider the torsional stiffness of the beam and column members.

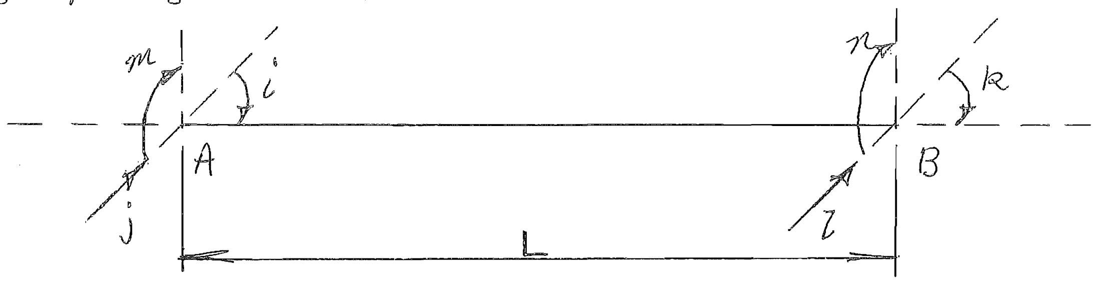
The weight of each beam was divided equally between adjacent columns and lumped at the joints with the column. Half the weight of each column was lumped at the top of the column. This gave sixteen masses each associated with a lateral co-ordinate.

7.3 ELASTIC ANALYSIS

The response of the structure to earthquake loading was predicted by determining the normal mode properties and summing the response of each mode as given by a response spectrum.

The normal mode properties were determined by iteration of the lateral flexibility matrix which was determined by the inversion of the stiffness matrix. A computer program was written to assemble the stiffness matrix. This program was similar to the routine contained in the program "DYNEPRES" used to assemble the stiffness matrix, where the reference numbers for the end deformations associated with the member are read with the member data and this enables the member stiffness coefficients to be calculated and added to the appropriate position in the frame stiffness matrix. The assembly routine must cope with the possibility of a hinge at either end and in addition the effect of a torsional moment at both ends and the inclined pier members.

The equilibrium equations for a typical beam AB shown in the diagram, are given below:



Where the letters in the diagram are used to distinguish actions and their corresponding deformations.

1. Then with no hinges present:

$$\begin{Bmatrix} M_i \\ F_j \\ M_k \\ F_l \\ M_m \\ M_n \end{Bmatrix} = \begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & 0 \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & 0 \\ & & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & 0 \\ & & & \frac{12EI}{L^3} & 0 & 0 \\ & & & & \frac{GJ}{L} & -\frac{GJ}{L} \\ & \text{symmetrical} & & & & \frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \\ \delta_l \\ \theta_m \\ \theta_n \end{Bmatrix} \quad (1)$$

where M denotes an action which is a bending moment,

F denotes an action which is a force,

θ denotes a deformation which is a rotation,

δ denotes a deformation which is a displacement,.

Subscripts are used to distinguish individual actions and deformations.

G, E, I and L are as previously defined.

J is the polar moment of inertia of the section.

$J = \dfrac{D^4}{32}$ for a circular section where D is the diameter.

$J = k_1 b^3 D$ for a rectangular section, b being the smaller side, D the larger side, k_1 being determined from tables prepared by Timoshenko[27].

2. With a hinge at end A the equilibrium equations become:

$$\begin{Bmatrix} M_i \\ F_j \\ M_k \\ F_l \\ M_m \\ M_n \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{3EI}{L^3} & \frac{3EI}{L^2} & \frac{-3EI}{L^3} & 0 & 0 \\ & & \frac{3EI}{L} & \frac{-3EI}{L^2} & 0 & 0 \\ & & & \frac{3EI}{L^3} & 0 & 0 \\ & & & & \frac{GJ}{L} & -\frac{GJ}{L} \\ & \text{symmetrical} & & & & \frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \\ \delta_l \\ \theta_m \\ \delta_n \end{Bmatrix} \qquad (2)$$

Where θ_i is the rotation about the vertical axis of joint A of the frame, as before, and not the rotation of end A of member AB

The formation of a plastic hinge is assumed not to affect the torsional stiffness about the longitudinal axis of the member.

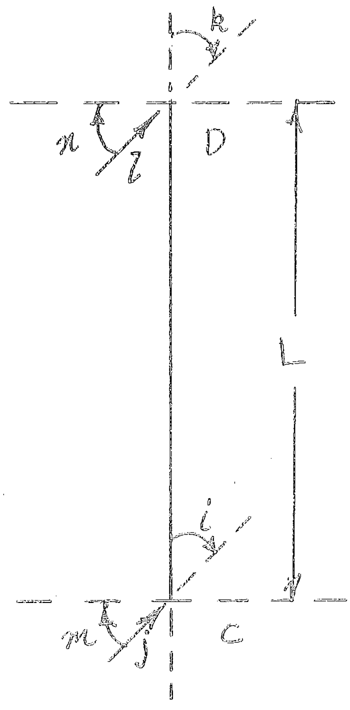
3. With a hinge at end B the equilibrium equations for the member AB become:

$$\begin{Bmatrix} M_i \\ F_j \\ M_k \\ F_l \\ M_m \\ M_n \end{Bmatrix} = \begin{bmatrix} \frac{3EI}{L} & \frac{3EI}{L^2} & \cdot & \frac{-3EI}{L^2} & \cdot & \cdot \\ & \frac{3EI}{L^3} & \cdot & -\frac{3EI}{L^3} & \cdot & \cdot \\ & & \cdot & \cdot & \cdot & \cdot \\ & & & \frac{3EI}{L^3} & \cdot & \cdot \\ & & & & \frac{GJ}{L} & -\frac{GJ}{L} \\ \text{symmetrical} & & & & & \frac{GJ}{L} \end{bmatrix} \cdot \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \\ \delta_l \\ \theta_m \\ \theta_n \end{Bmatrix} \quad (3)$$

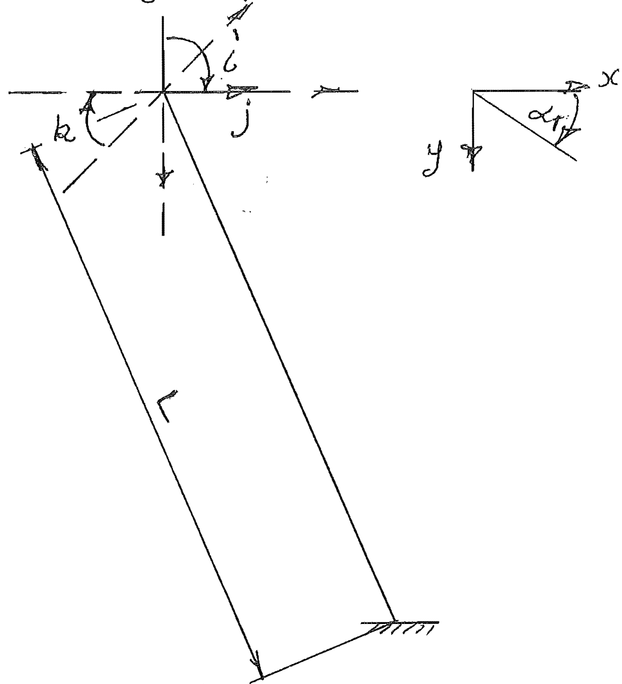
4. With hinges at both ends A and B the equilibrium equations become:

$$\begin{Bmatrix} M_i \\ F_j \\ M_k \\ F_l \\ M_m \\ M_n \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & \frac{GJ}{L} & -\frac{GJ}{L} \\ \text{symmetrical} & & & & & \frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \\ \delta_l \\ \theta_m \\ \theta_n \end{Bmatrix} \quad (4$$

The equilibrium equations for a typical column member CD shown in the diagram are identical to (1) with no hinges present and identical to (2) with a hinge at the base.



With the inclined members it is only necessary to consider the equilibrium of one joint as the other end is fully fixed. For a pier member inclined at angle α_1 to the horizontal x axis:



There are three unknown deformations and the member equilibrium equations may be written:

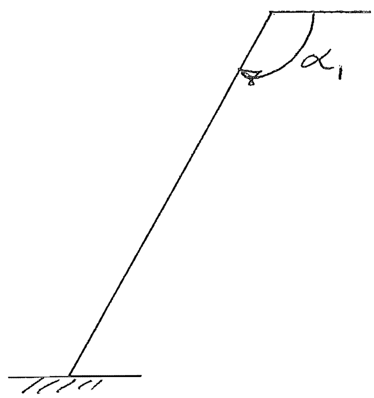
$$\begin{Bmatrix} M_i \\ f_j \\ M_k \end{Bmatrix} = \begin{bmatrix} \frac{4EI}{L} & -S\frac{6EI}{L^2} & \circ \\ -S\frac{6EI}{L^2} & (C^2\frac{EA}{L} + S^2\frac{12EI}{L^3}) & \circ \\ \circ & \circ & S\frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \theta_i \\ \delta_j \\ \theta_k \end{Bmatrix}$$

where E, I, L, G, J, and A are as previously defined.

$$S = \sin\alpha_1$$

$$C = \cos\alpha_1$$

α_1 is the angle measured to the x-axis, thus for the left-hand member, α_1 is obtuse.



After the stiffness matrix has been assembled and inverted, the lateral flexibility matrix is formed by picking out the appropriate coefficients from the flexibility matrix. A program "BRFLAT" to accomplish this is shown in List 13.

The normal mode properties were found by iteration using a program similar to that written by Donald[18]. The values of the mass lumped at the co-ordinate points are given in Table 7.1. The maximum elastic response of the bridge was predicted by finding the response of each mode using Skinner's response curve for 10% critical damping and taking the root mean square of the modal responses.

The normal mode properties and the predicted response are listed in Table 7.1. The notation of the lateral deformations used in Table 7.1 corresponds to that shown in Figure 7.4. The displaced shape of the deck beam in the first three modes is shown in Figure 7.2.

The maximum response of the bridge to the 1940 El Centro N-S earthquake was found by integrating the first three modal equations of motion and summing the response of each mode at the end of each step interval and also by integrating the sixteen equations of motion for the lumped masses. The two integrations give practically identical answers and are compared with the displacements found by the root mean square of the modal responses in Figure 7.2.

The member actions associated with the maximum elastic displacements under the El Centro record were found by determining the statically equivalent forces required to produce these displacements. These forces were found by inverting the lateral flexibility matrix, using the program "BRKLAT" given in List 14, to give the lateral stiffness matrix and then multiplying this by the maximum lateral displacements. The rotations produced by these equivalent forces were found by multiplying the appropriate sub-matrix of the flexibility matrix by the forces.

The process then consists of the following steps:

$$\left\{P_{eq}\right\} = \left[K_{LAT}\right] \cdot \left\{\delta_{max}\right\}$$

where $\left\{\delta_{max}\right\}$ is a vector of the maximum lateral displacements,

$\left\{P_{eq}\right\}$ is the vector of equivalent lateral forces,

$\left[K_{LAT}\right]$ is the lateral stiffness matrix.

Then

$$\left\{\theta\right\} = \left[F_{12}\right] . \left\{P_{eq}\right\}$$

where $\left\{\theta\right\}$ is the vector of rotations of the bridge,

$\left[F_{12}\right]$ is a sub-matrix of the flexibility matrix, when it is partitioned so that the terms associated with the rotations are separated from those associated with the displacements, as follows:

$$\left\{\begin{matrix} \theta \\ \\ \delta \end{matrix}\right\} = \begin{bmatrix} F_{11} & F_{12} \\ \\ F_{21} & F_{22} \end{bmatrix} . \left\{\begin{matrix} M \\ \\ P \end{matrix}\right\}$$

where δ is the vector of the lateral displacements,

M is a vector of applied moments, usually null,

P is a vector of the lateral forces,

F_{11}, F_{12}, F_{21} and F_{22} being sub-matrices of the flexibility matrix. F_{22} being the lateral flexibility matrix $\left[F_{LAT}\right]$.

A program "BRDEF" to carry out these operations is shown in List 15. Once the bridge deformations have been found the individual member actions may be found by setting up the member stiffness matrices and multiplying by the appropriate deformations. A program "BRMACT" to do this is shown in List 16.

The member actions caused by a code type loading, with lateral loads determined by using a seismic coefficient of 0.15 at the deck level and 0.05 at the tie beam level, were also found by using the programs in Lists 15 and 16. A factor of roughly 4.5 was needed to reduce the predicted elastic response to the code response. The moments in the deck beam caused by the code-type loading are compared with the maximum moments assuming elastic response to the El Centro record and with the ultimate moments in Figure 7.7.

7.4 ELASTO-PLASTIC ANALYSIS

A computer program "BRIDGE" was written to determine the elasto-plastic response of the bridge to the El Centro N-S record, assumed to be applied transversely to the structure. The program is similar to the program "DYNEPRES" except that the stiffness matrix assembly procedure was modified to incorporate the same effects as detailed in the Elastic Analysis section. The member stiffness matrices are identical to those detailed in that section and they are used to form the incremental action - deformation relationships for the program. A listing of the program is given in List 12.

The bridge was run with this program initially keeping all the members elastic by using high yield moments and it was found that this elastic response was practically identical to that found previously. The bridge was then run allowing plastic hinges to form in the deck beam if the ultimate moments were reached.

The ultimate moments of the beams were assumed to be half the maximum moment reached assuming elastic behaviour, except that no beam was allowed to have a yield moment less than 35×10^6 in.lb, half the greatest ultimate moment. This procedure meant that the ultimate moments were set at approximately twice the moments caused by code loading. This is in line with current New Zealand practice.

The variation of the lateral displacement of pier number 10 with time is given in Figure 7.5. (The pier numbers are given in Figure 7.1.) The maximum displacements of the piers are compared with the maximum values assuming elastic behaviour and with the displaced shape caused by code loading in Figure 7.3. It can be seen that the plastic action reduces the maximum displacements a little, but the displaced shape is essentially similar.

The variation of the bending moment with time at the end of a typical deck beam member, that between piers 9 and 10, is shown in Figure 7.6 together with the growth of plastic deformation for the same section.

The plastic deformations for the deck beam are given in the form of the member ductility ratio in Figure 7.8 and are compared with the ratio of the maximum moment assuming elastic behaviour to the ultimate moment for the same section. There is a definite tendency for the peaks in the moment ratio curve to be accentuated in the ductility ratio plot, particularly where there was a relatively large elastic deflection. The maximum ductility ratio was 4.7.

TABLE 7.1

NORMAL MODE PROPERTIES

Mode 1

Frequency = 1.87 c.p.s. Period = 0.534

Amplification Factor = 0.62

Mass	Displacement Ratios	1g Displacements inches	1g Shears Kips
16	.0314	.0098	0.58
15	-.0932	-.0291	-1.92
14	-.5204	-.1624	-10.93
13	-.8123	-.2536	-18.45
12	-.6162	-.1923	-14.78
11	.0219	.0068	.53
10	.7236	.2259	17.36
9	1.0000	.3121	23.99
8	.6820	.2129	16.36
7	.1086	.0339	2.61
6	-.1832	-.0572	-3.97
5	-.1936	-.0604	-3.86
4	-.0818	-.0256	-1.53
3	.0323	.0101	.77
2	.0586	.0183	1.44
1	.0312	.0098	.75

Mode 2

Frequency = 2.36 Period = 0.424

Amplification Factor = 0.71

Mass	Displacement Ratios	1g Displacements inches	1g Shears Kips
16	-.0379	-.089	-8.4
15	.1380	.325	34.0
14	.6978	1.644	175.5
13	1.0000	2.356	271.9
12	.7035	1.657	202.0
11	.2847	.671	81.8
10	.4747	1.118	136.3
9	.7245	1.707	208.1
8	.5504	1.296	158.1
7	.0907	.214	26.1
6	-.1824	-.430	-47.3
5	-.2043	-.481	-48.8
4	-.0906	-.213	-20.3
3	.0210	.050	6.0
2	.0416	.098	12.2
1	.0237	.056	6.8

TABLE 7.1 continued

Mode 3

Frequency = 3.165 Period = 0.316

Amplification Factor = 0.72

Mass	Displacement Ratios	1g Displacements inches	1g Shears Kips
16	-.0006	-.0010	-.17
15	.0049	.0075	1.40
14	.0198	.0297	5.71
13	.0227	.0340	7.07
12	.0166	.0249	5.47
11	.0491	.0736	16.15
10	.1472	.2205	48.39
9	.1606	.2406	52.80
8	.1064	.1593	34.97
7	.2792	.4182	91.78
6	.8608	1.2890	255.30
5	1.0000	1.4970	273.20
4	.4881	.7311	124.80
3	.0039	.0059	1.28
2	.0050	.0076	1.71
1	.0012	.0019	.40

PREDICTED ELASTIC RESPONSE

Coordinate	Lumped Mass Kips	Displacement inches	Force Kips
16	164.9	.064	6.0
15	183.9	.232	24.3
14	187.7	1.177	125.5
13	202.9	1.688	194.4
12	214.3	1.188	144.5
11	214.3	.482	59.5
10	214.3	.826	104.0
9	214.3	1.246	154.1
8	214.3	.942	116.0
7	214.3	.341	69.2
6	193.4	.986	188.5
5	178.2	1.141	201.5
4	166.8	.553	91.9
3	213.2	.036	4.4
2	218.9	.071	8.9
1	213.2	.040	4.9

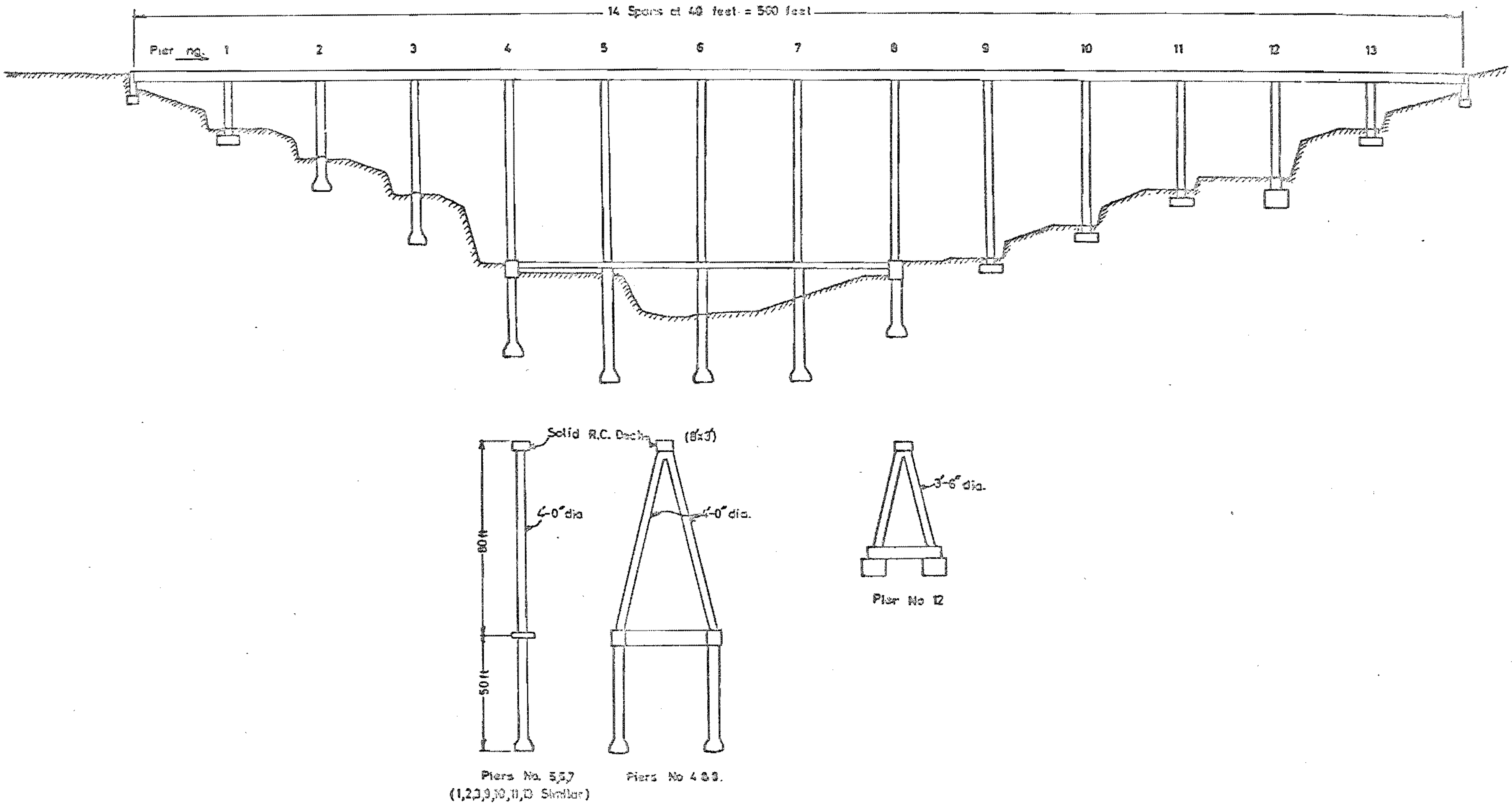

14 Spans at 40 feet = 560 feet
Pier no. 1 2 3 4 5 6 7 8 9 10 11 12 13
Solid R.C. Deck
(8'x3')
4'-0" dia
80 ft
50 ft
4'-0" dia.
3'-6" dia.
Pier No 12
Piers No. 5,6,7
(1,2,3,9,10,11,13 Similar)
Piers No 4 & 8.

FIGURE 7·1

MODE SHAPES and PREDICTED RESPONSE.

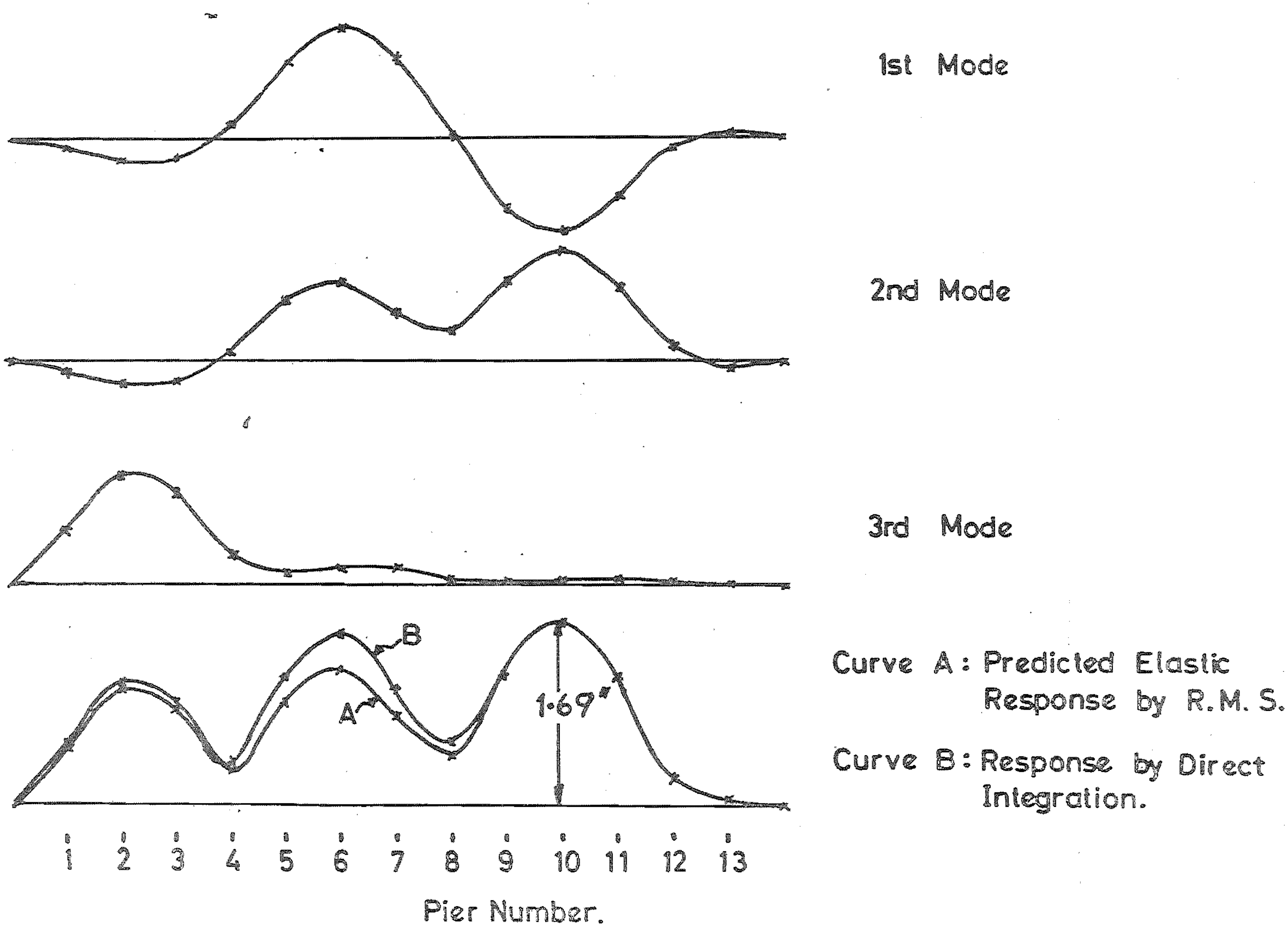


FIGURE 7·2

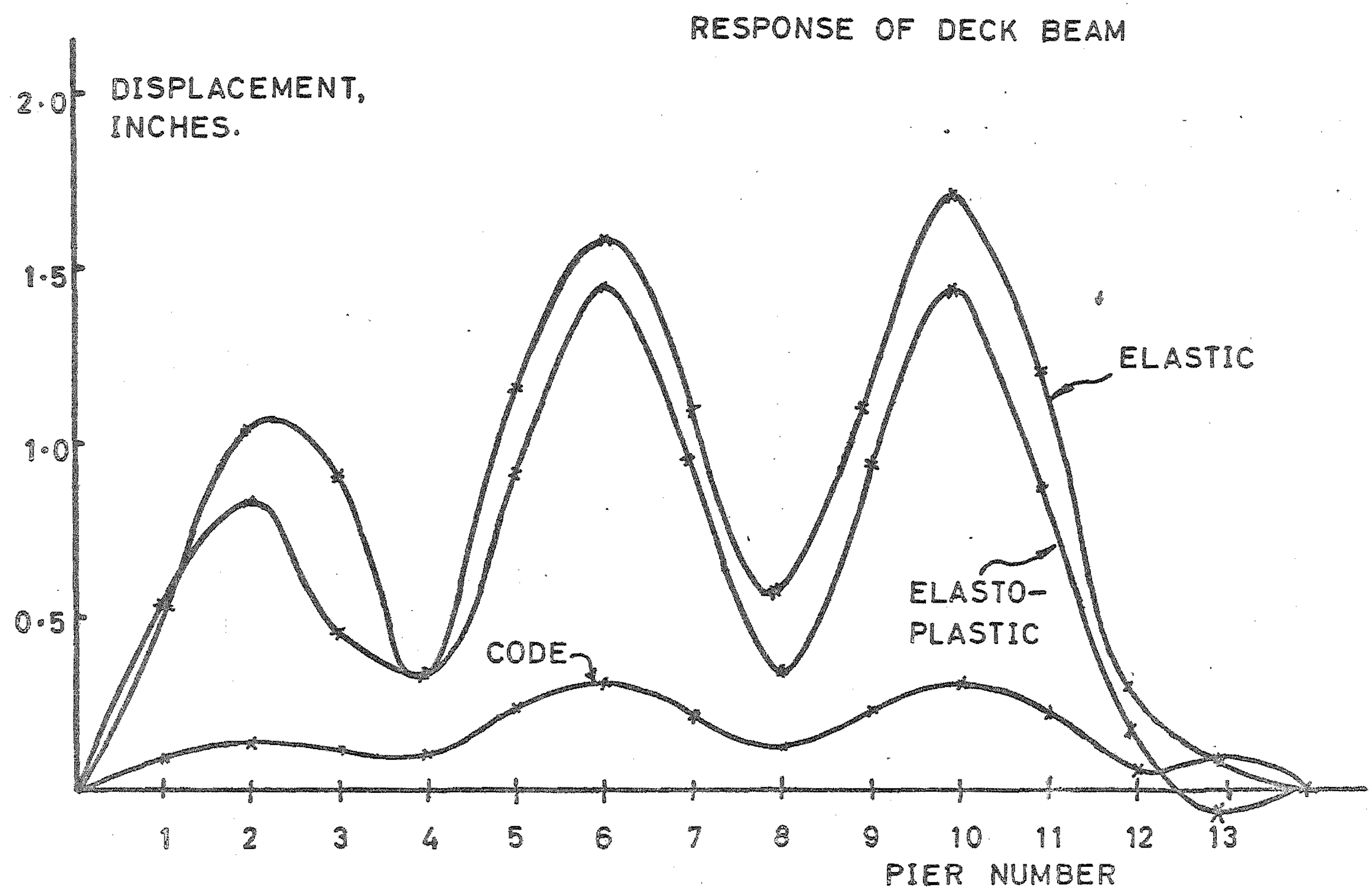

RESPONSE OF DECK BEAM
DISPLACEMENT, INCHES.
2·0
1·5
1·0
0·5
ELASTIC
ELASTO-PLASTIC
CODE
1
2
3
4
5
6
7
8
9
10
11
12
13
PIER NUMBER

FIGURE 7·3

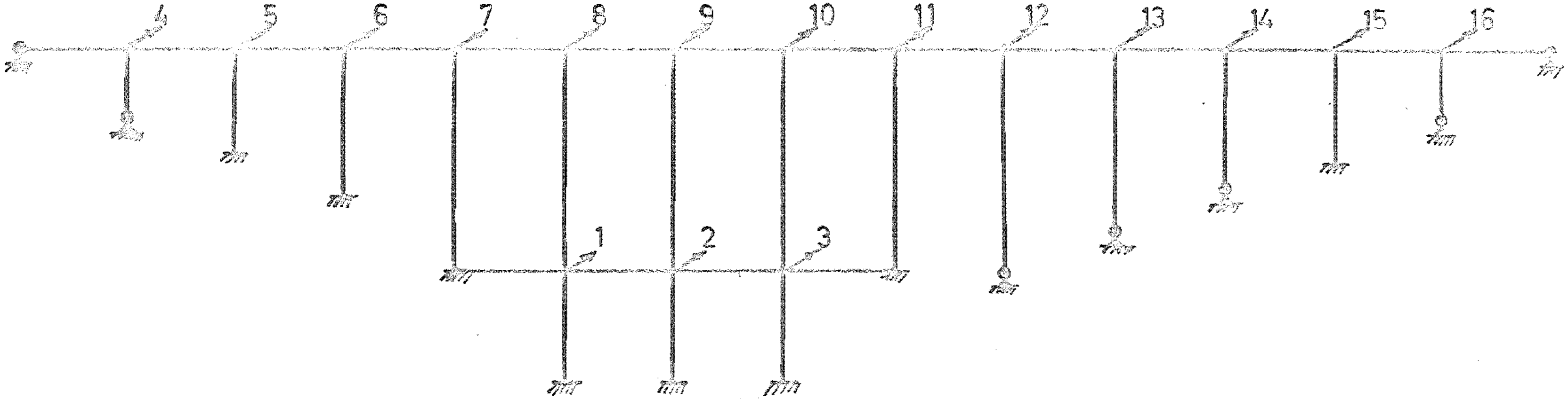


FIGURE 7·4

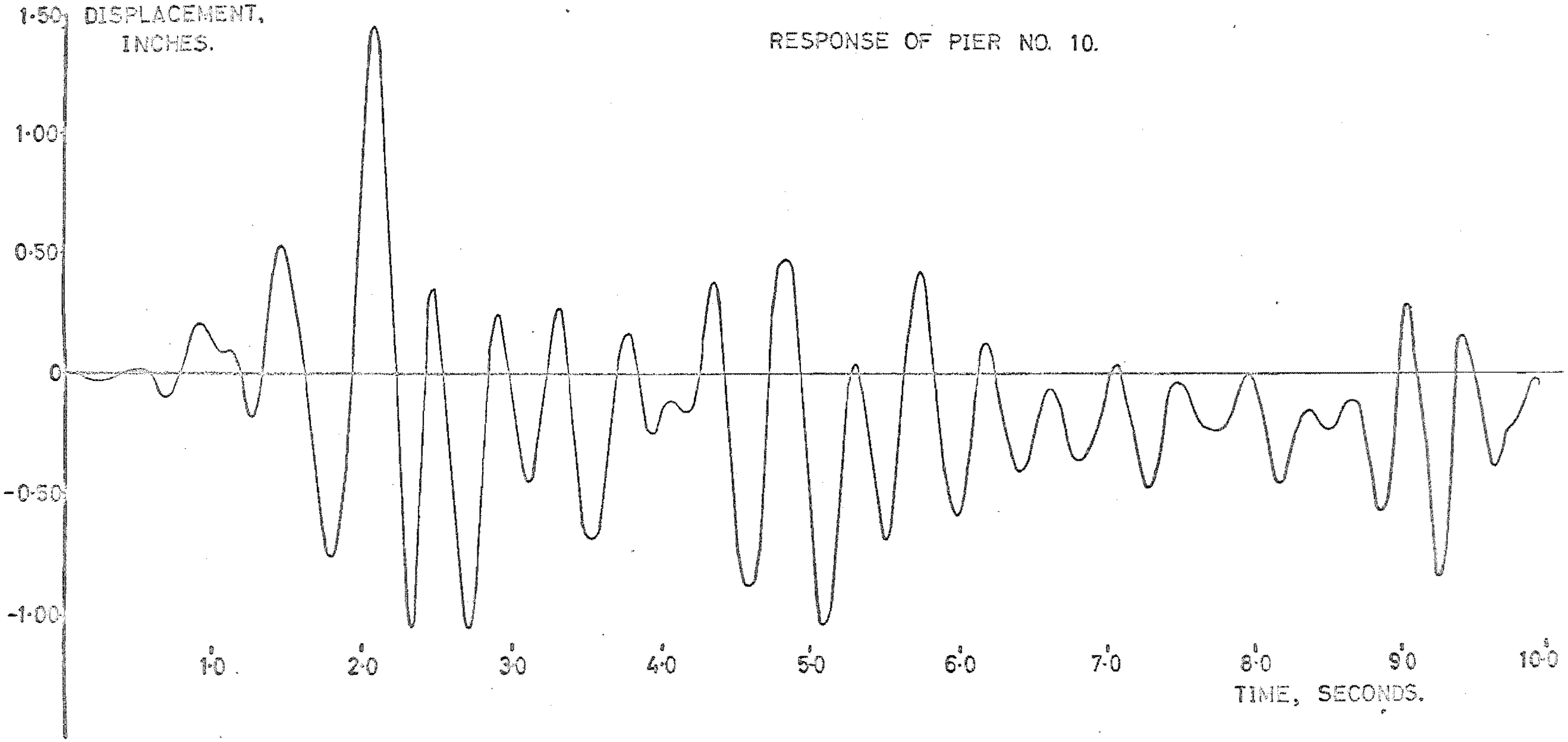


FIGURE 7·5

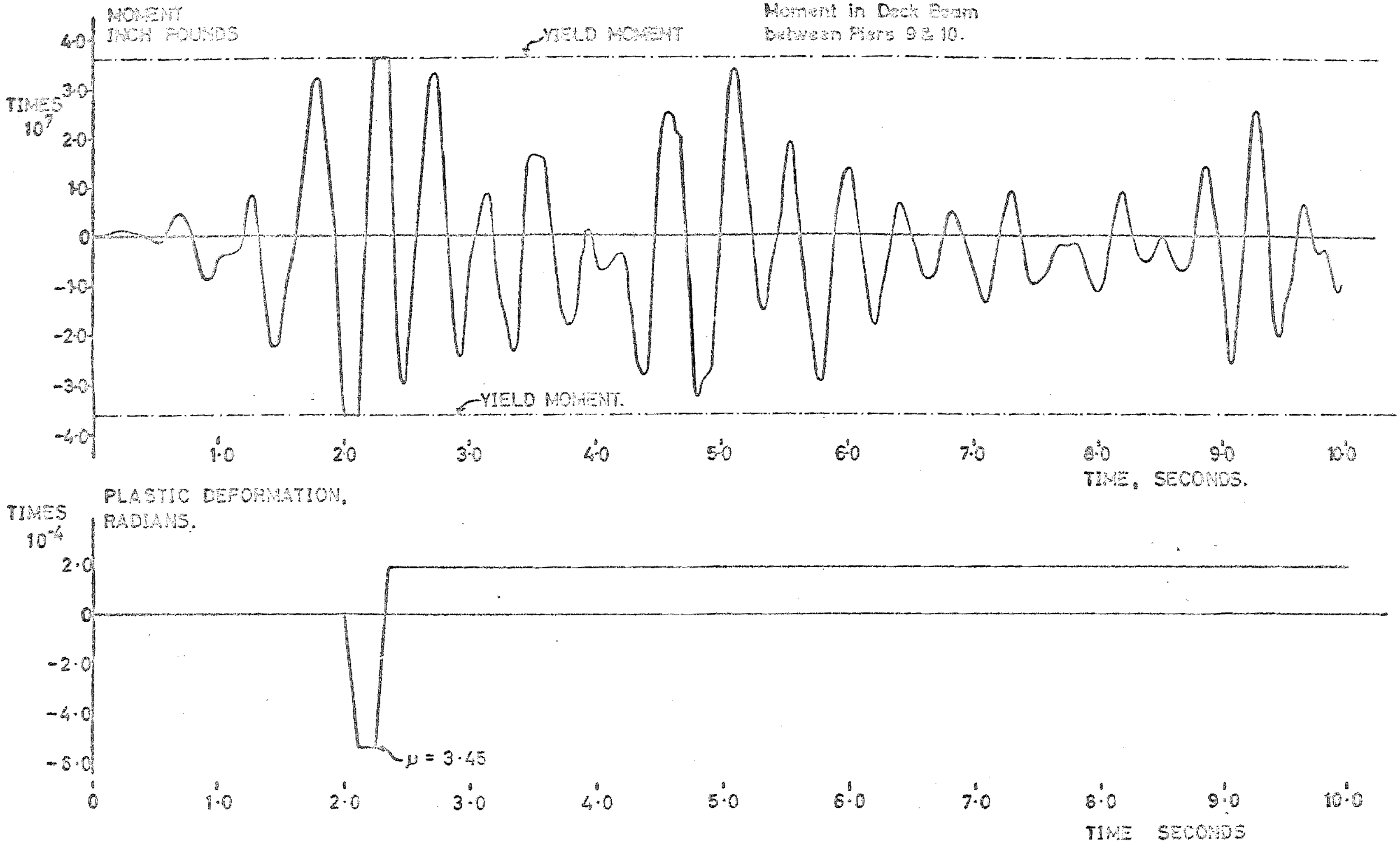

MOMENT
INCH POUNDS
TIMES
10^7
YIELD MOMENT
Moment in Deck Beam
between Piers 9 & 10.
4·0
3·0
2·0
1·0
0
-1·0
-2·0
-3·0
-4·0
YIELD MOMENT.
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME, SECONDS.
PLASTIC DEFORMATION,
RADIANS.
TIMES
10^{-4}
2·0
0
-2·0
-4·0
-6·0
μ = 3·45
0
1·0
2·0
3·0
4·0
5·0
6·0
7·0
8·0
9·0
10·0
TIME SECONDS

FIGURE 7·6

MOMENTS IN DECK BEAM.

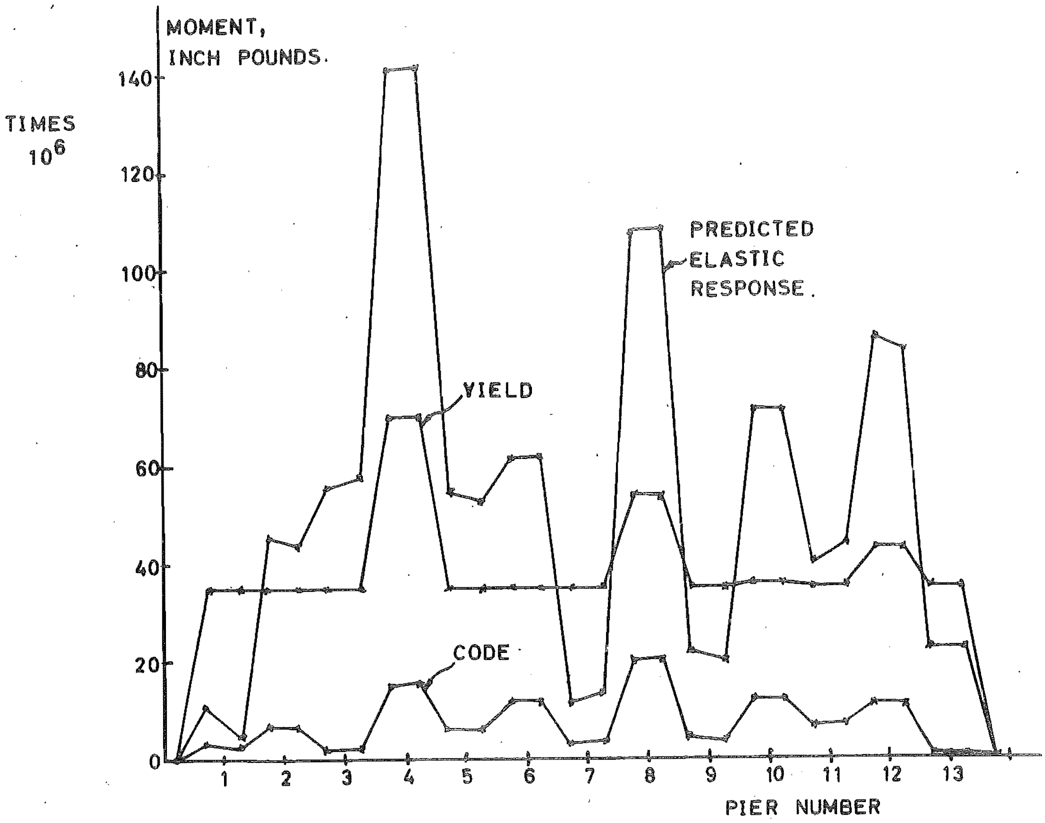


FIGURE 7·7

DUCTILITY REQUIRED IN DECK BEAM.

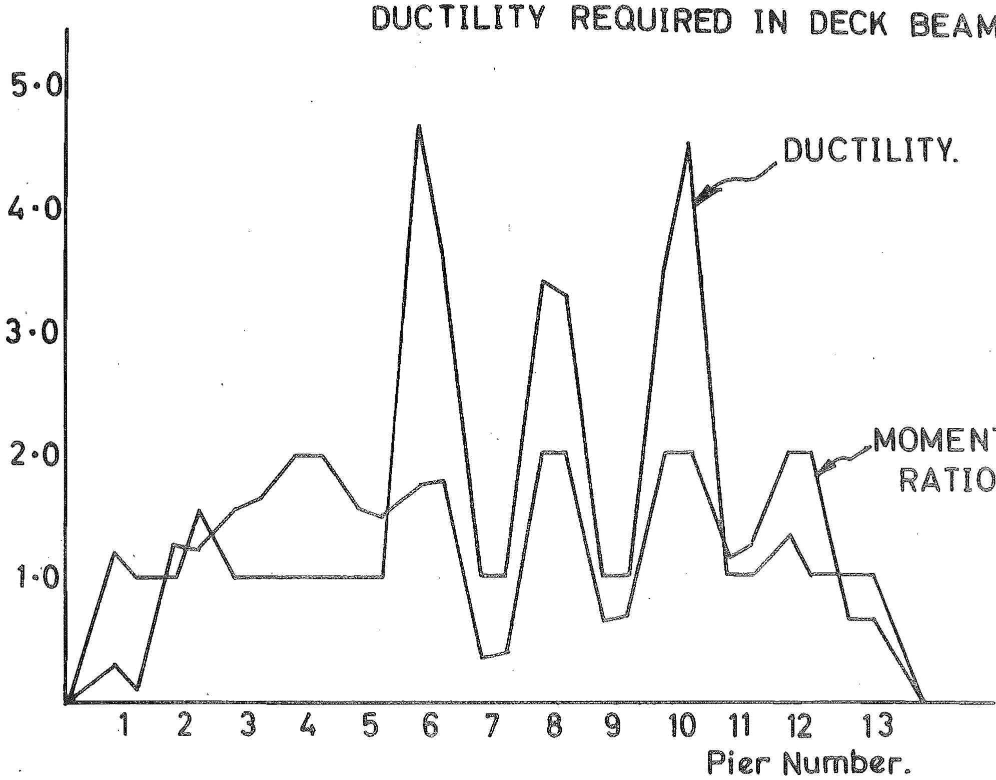


FIGURE 7·8

```
C       PROGRAM ''BRIDGE''    LIST 12.                        PAGE 1
C       LANGUAGE - FORTRAN IV FOR IBM 360/40
        DIMENSION T(5),G(5),TSD(48),SD(49),VEL(16),ACC(16),S(48,48),
       1C(16,16),AA(16),BB(16),W(16),DR(48),IS(40),JS(40),KS(40),LS(40),
       2MS(40),NW(40),P1(40),Q1(40),PJ(40),YM1(40),YM2(40),IH1(40),IP1(20)
       3,IH2(40),PD1(20),PD2(20),PDM1(20),PDM2(20),R1(40),R2(40),IP2(20)
        READ(1,30)N,NS,NB,M4,M3,M5,KP,DT,TR,E,GG
   30   FORMAT(7I3,4E10.4)
        SD(N&1)=0.
        KC=KP
        M6=M4-M5
        NB2=NB&2
        NB1=NB&1
        ISQ=1
        MPR=1
        DO171M=1,M3
        PD1(M)=0.
  171   PD2(M)=0.
        WRITE(3,330)N,NS,NB,M4,M3,M5,KP,DT,TR,E,GG
  330   FORMAT(1X,7I4,4E13.4///)
C       READ AND STORE MEMBER DATA
        DO28M=1,M4
        READ(1,31)IS(M),JS(M),KS(M),LS(M),MS(M),NW(M),P1(M),Q1(M),PJ(M),
       1YM1(M),YM2(M),IH1(M),IH2(M),R1(M),R2(M)
   31   FORMAT(6I4,5E6.0,2I3,2E10.0)
        WRITE(3,331)IS(M),JS(M),KS(M),LS(M),MS(M),NW(M),P1(M),Q1(M),PJ(M),
       1YM1(M),YM2(M),IH1(M),IH2(M),R1(M),R2(M)
  331   FORMAT(1X,6I3,5E13.4,2I3,2E13.4)
        YM1(M)=YM1(M)*1.E3
        YM2(M)=YM2(M)*1.E3
        P1(M)=P1(M)*E*20736./Q1(M)
   28   PJ(M)=PJ(M)*20736.
        WRITE(3,341)
        DO29I=1,NS
        READ(1,32)(C(I,J),J=1,NS)
        WRITE(3,332)(C(I,J),J=1,NS)
   29   WRITE(3,340)
   32   FORMAT(5E14.8)
  332   FORMAT(1X,5E18.8)
  340   FORMAT(2X)
        WRITE(3,341)
  341   FORMAT(//)
        READ(1,33)(W(I),I=1,NS)
   33   FORMAT(8E10.4)
        WRITE(3,333)(W(I),I=1,NS)
  333   FORMAT(1X,8E13.4/)
        WRITE(3,341)
        DO40I=1,NS
   40   W(I)=W(I)/386.4
        DO41I=1,M3
        PDM1(I)=0.
        PDM2(I)=0.
        IP1(I)=IH1(I)
   41   IP2(I)=IH2(I)
        DO42I=1,N
   42   TSD(I)=0.
        DO43I=1,NS
   43   VEL(I)=0.
        TM=0.
        XMAX=0.
        MSK=1
```

```
C       PROGRAM ''BRIDGE''    LIST 12.                        PAGE 2
        ICC=1
        READ(1,35)ISC,(T(I),G(I),I=2,5)
   35   FORMAT(I3,4(F8.4,F9.6))
        IF(ICC-ISC)44,45,44
   45   M2=2
        PP=G(M2)*386.4
        DG2=(G(3)-G(2))/(T(3)-T(2))*DT*386.4
        DO46I=1,NS
   46   ACC(I)=-G(2)*386.4
   47   KC=KC&1
        IF(KC-KP)356,357,357
  357   KC=0
        WRITE(3,340)
        WRITE(3,334)TM,(TSD(I),I=3,N,3),ISQ
  334   FORMAT(1X,9E13.4,/14X,8E13.4,10X,I4)
        WRITE(3,343)R2(4),R1(5),R2(6),R1(7),R2(8),R1(9),
       1R2(10),R1(11),R2(12),R1(13)
        WRITE(3,343)PD2(4),PD1(5),PD2(6),PD1(7),PD2(8),PD1(9),
       1PD2(10),PD1(11),PD2(12),PD1(13)
  343   FORMAT(1X,10E13.4)
        WRITE(2,342)TM,TSD(27),TSD(39),R2(4),R1(5),R2(6),R1(7),ISQ
        WRITE(2,342)TM,R2(8),R1(9),R2(10),R1(11),R2(12),R1(13),ISQ,ISQ
  342   FORMAT(7E10.3,2I5)
        ISQ=ISQ&1
  356   IF(TM&DT-T(M2&1))48,48,49
   48   TM=TM&DT
        DG=DG2
        PP=PP&DG
        GO TO 50
   49   IF(M2-4)80,81,81
   80   M2=M2&1
        GO TO 83
   81   M2=1
        G(1)=G(5)
        T(1)=T(5)
        ICC=ICC&1
        READ(1,35)ISC,(T(I),G(I),I=2,5)
        IF(ICC-ISC)44,83,44
   44   WRITE(3,337)ICC
  337   FORMAT(1X,19HCARDS OUT OF ORDER.I5)
        STOP
   83   TT=TM&DT-T(M2)
        TS=T(M2&1)-T(M2)
        IF(TS)49,49,84
   84   GS=(G(M2&1)-G(M2))*386.4
        GB=GS*TT/TS
        DG2=GS*DT/TS
        TM=TM&DT
        DG=G(M2)*386.4&GB-PP
        PP=G(M2)*386.4&GB
   50   DO85I=1,NS
        AA(I)=-6.0*VEL(I)/DT-3.0*ACC(I)
   85   BB(I)=-3.0*VEL(I)-0.5*DT*ACC(I)
        DO86I=1,NS
        DO82K=1,NB1
        L=(I-1)*NB2&K
   82   DR(L)=0.
        GS=0.
        DO87J=1,NS
   87   GS=GS&C(I,J)*BB(J)
```

```
      K=I*NB2
   86 DR(K)=-W(I)*DG-W(I)*AA(I)-GS
      GO TO (88,89),MSK
C     SET UP FRAME STIFFNESS MATRIX
   88 WRITE(3,341)
      WRITE(3,333)TM
      WRITE(3,106)(IH1(I),IH2(I),I=1,M3)
  106 FORMAT(1X,36I3)
      WRITE(3,340)
      DO140I=1,N
      DO140J=1,N
  140 S(I,J)=0.
      DO123M=1,M6
      I=IS(M)
      J=JS(M)
      K=KS(M)
      L=LS(M)
      MM=MS(M)
      NN=NW(M)
      P=P1(M)
      QQ=GG*PJ(M)/Q1(M)
      IF(IH1(M)-1)3,4,4
    3 IF(IH2(M)-1)5,8,8
    4 IF(IH2(M)-1)6,27,27
    5 R=4.0
      Q=6.0/Q1(M)
      GO TO 7
    8 K=N&1
      GO TO 26
    6 I=N&1
   26 R=3.0
      Q=3.0/Q1(M)
    7 IF(I-N)9,9,10
    9 S(I,I)=S(I,I)&R*P
      IF(J-N)11,11,12
   11 S(I,J)=S(I,J)&Q*P
      S(J,I)=S(I,J)
   12 IF(K-N)13,13,14
   13 S(I,K)=S(I,K)&2.0*P
      S(K,I)=S(I,K)
   14 IF(L-N)15,15,16
   15 S(I,L)=S(I,L)-Q*P
      S(L,I)=S(I,L)
   10 IF(L-N)17,17,16
   17 S(L,L)=S(L,L)&Q*Q*P*.33333333
      IF(J-N)18,18,19
   18 S(J,L)=S(J,L)-Q*Q*P*.33333333
      S(L,J)=S(J,L)
   19 IF(K-N)20,20,16
   20 S(K,L)=S(K,L)-Q*P
      S(L,K)=S(K,L)
   16 IF(J-N)21,21,22
   21 S(J,J)=S(J,J)&Q*Q*P*.33333333
      IF(K-N)23,23,27
   23 S(J,K)=S(J,K)&Q*P
      S(K,J)=S(J,K)
   22 IF(K-N)24,24,27
   24 S(K,K)=S(K,K)&R*P
   27 IF(MM-N)120,120,121
  120 S(MM,MM)=S(MM,MM)&QQ
```

LIST 12


```
      IF(NN-N)122,122,123
  122 S(MM,NN)=S(MM,NN)-QQ
      S(NN,MM)=S(MM,NN)
  121 IF(NN-N)124,124,123
  124 S(NN,NN)=S(NN,NN)&QQ
  123 CONTINUE
      DO125MM=1,M5
      M=MM&M6
      I=IS(M)
      J=JS(M)
      K=KS(M)
      Q=Q1(M)
      ANG=3.1416/180.*R2(M)
      AR=R1(M)
      S(I,I)=S(I,I)&4.0*P1(M)
      S(I,K)=S(I,K)-6.0*SIN(ANG)*P1(M)/Q1(M)
      S(K,I)=S(I,K)
      S(K,K)=S(K,K)&12.0*(SIN(ANG))**2*P1(M)/(Q*Q)
     1&E*AR*(COS(ANG))**2/Q
  125 S(J,J)=S(J,J)&GG*PJ(M)*SIN(ANG)/Q
      GO TO (150,151),MPR
  150 DO 152 I=1,N
      WRITE(3,153)(S(I,J),J=1,N)
  152 WRITE(3,340)
  153 FORMAT(1X,8E13.4)
      MPR=2
  151 DO 90I=1,NS
      K=I*NB2
      K=I*NB2
      S(K,K)=S(K,K)&6.0*W(I)/(DT*DT)
      DO90J=1,NS
      L=J*NB2
   90 S(K,L)=S(K,L)&3.0*C(I,J)/DT
C     INVERTS (S) IN SITU. ORDER NXN.
      DO101I=1,N
      T2=S(I,I)
      IF(T2)350,351,350
  351 WRITE(3,339)
  339 FORMAT(1X,15HSINGULAR MATRIX////)
      STOP
  350 S(I,I)=1.0
      DO102J=1,N
  102 S(I,J)=S(I,J)/T2
      DO101L=1,N
      IF(L-I)103,101,103
  103 T2=S(L,I)
      S(L,I)=0.
      DO104J=1,N
  104 S(L,J)=S(L,J)-T2*S(I,J)
  101 CONTINUE
   89 DO91I=1,N
      GS=0.
      DO92J=1,N
   92 GS=GS&S(I,J)*DR(J)
      SD(I)=GS
   91 TSD(I)=TSD(I)&SD(I)
      DO93I=1,NS
      J=I*(NB&2)
      VEL(I)=VEL(I)&3.0*SD(J)/DT&BB(I)
   93 ACC(I)=ACC(I)&6.0*SD(J)/(DT*DT)&AA(I)
```

```
      IF(ABS(TSD(27))-ABS(XMAX))112,112,113
  113 XMAX=TSD(27)
      TMAX=TM
  112 DO78M=1,M3
      I=IS(M)
      J=JS(M)
      K=KS(M)
      L=LS(M)
      P=P1(M)
      IF(IH1(M)-1)51,52,52
   51 IF(IH2(M)-1)53,54,54
   52 IF(IH2(M)-1)55,56,56
   54 R1(M)=R1(M)&(3.0*SD(I)&3.0/Q1(M)*(SD(J)-SD(L)))*P
      Q=SD(K)&0.5*SD(I)-1.5*(SD(L)-SD(J))/Q1(M)
      MM=1
   58 IF(R2(M)*Q)60,62,62
   60 IH2(M)=0
      GO TO 66
   62 PD2(M)=PD2(M)&Q
      IF(ABS(PD2(M))-ABS(PDM2(M)))66,66,65
   65 PDM2(M)=PD2(M)
   66 IF(ABS(R1(M))-YM1(M))63,63,76
   76 IH1(M)=1
      R1(M)=.9999*YM1(M)*R1(M)/ABS(R1(M))
   63 GO TO (78,67),MM
   55 R2(M)=R2(M)&(3.0*SD(K)&3.0/Q1(M)*(SD(J)-SD(L)))*P
      Q=SD(I)&0.5*SD(K)-1.5*(SD(L)-SD(J))/Q1(M)
   68 IF(R1(M)*Q)72,74,74
   72 IH1(M)=0
      GO TO 79
   74 PD1(M)=PD1(M)&Q
      IF(ABS(PD1(M))-ABS(PDM1(M)))79,79,75
   75 PDM1(M)=PD1(M)
   79 IF(ABS(R2(M))-YM2(M))78,78,170
  170 IH2(M)=1
      R2(M)=.9999*YM2(M)*R2(M)/ABS(R2(M))
      GO TO 78
   56 Q=SD(K)-(SD(L)-SD(J))/Q1(M)
      MM=2
      GO TO 58
   67 Q=SD(I)-(SD(L)-SD(J))/Q1(M)
      GO TO 68
   53 R1(M)=R1(M)&(4.0*SD(I)&2.0*SD(K)&6.0/Q1(M)*(SD(J)-SD(L)))*P
      R2(M)=R2(M)&(2.0*SD(I)&4.0*SD(K)&6.0/Q1(M)*(SD(J)-SD(L)))*P
      IF(ABS(R1(M))-YM1(M))79,79,77
   77 IH1(M)=1
      R1(M)=.9999*YM1(M)*R1(M)/ABS(R1(M))
      GO TO 79
   78 CONTINUE
      IF(TM-TR&.5*DT)94,95,95
   94 MSK=2
      DO100I=1,M3
      IF(IH1(I)-IP1(I))198,99,198
   99 IF(IH2(I)-IP2(I))198,199,198
  198 MSK=1
  199 IP1(I)=IH1(I)
  100 IP2(I)=IH2(I)
      GO TO 47
   95 WRITE(3,333)XMAX,TMAX
      DO96I=1,M3
```



```
      U1=1.0&ABS(PDM1(I))*P1(I)*6.0/YM1(I)
      U2=1.0&ABS(PDM2(I))*P1(I)*6.0/YM2(I)
   96 WRITE(3,336)I,U1,U2
  336 FORMAT(1X,I3,2F10.2)
      WRITE(3,338)
  338 FORMAT(1X,////19HPROCESSING COMPLETE////)
      STOP
      END
```



```
      DIMENSION A(20,20)
    8 READ29,N
   29 FORMAT(I3)
      DO1I=1,N
      DO1J=1,N,5
    1 READ34,A(I,J),A(I,J&1),A(I,J&2),A(I,J&3),A(I,J&4)
   34 FORMAT(5E14.8)
      DO2I=1,N
      T=A(I,I)
      A(I,I)=1.0
      DO4J=1,N
    4 A(I,J)=A(I,J)/T
      DO2K=1,N
      IF(K-I)5,2,5
    5 T=A(K,I)
      A(K,I)=0.
      DO6J=1,N
    6 A(K,J)=A(K,J)-T*A(I,J)
    2 CONTINUE
      DO7I=1,N
      DO7J=1,N,5
    7 PUNCH34,A(I,J),A(I,J&1),A(I,J&2),A(I,J&3),A(I,J&4)
      STOP 9999
      GO TO 8
      END
```



```
      DIMENSION SD(50)
    1 READ30,N,MC,NS,NM,MS,E,G
   30 FORMAT(5I3,2E10.4)
      SD(50)=0.
      DO6I=1,48,8
    6 READ33,SD(I),SD(I&1),SD(I&2),SD(I&3),SD(I&4),SD(I&5),SD(I&6),
     1SD(I&7)
      DO2IK=1,MC
      READ31,I,J,K,L,MM,NN,P,Q,PJ,IH1,IH2
   31 FORMAT(6I3,3E10.4,2I3)
      P=P*E*20736./Q
      PJ=PJ*20736.
      IF(IH1-1)3,4,4
    3 R1=(4.0*SD(I)&2.0*SD(K)&6.0*(SD(J)-SD(L))/Q)*P
      R2=(2.0*SD(I)&4.0*SD(K)&6.0*(SD(J)-SD(L))/Q)*P
      R3=(R1&R2)/Q
      GO TO 5
    4 R2=(3.0*SD(K)&3.0*(SD(J)-SD(L))/Q)*P
      R3=R2/Q
      R1=0.
    5 R4=G*PJ*(SD(MM)-SD(NN))/Q
      R5=G*PJ*(SD(NN)-SD(MM))/Q
    2 PUNCH33,R1,R2,R3,R4,R5
   33 FORMAT(8E10.4)
      STOP 9999
      GO TO 1
      END
```

```
C     PROGRAM ''BRFLAT''    LIST 13.                          PAGE 1
C     PROGRAM ASSUMES LATERAL FORCES HAVE BEEN DENOTED
C     AS THE FIRST NS JOINT ACTIONS.
      DIMENSION A(48,48)
    1 READ30,N,MC,NS,NM,MS,E,G
   30 FORMAT(5I3,2E10.4)
C     INITIALISE ELEMENTS OF(K)
      DO3I=1,N
      DO3J=1,N
    3 A(I,J)=0.
C     READ ELEMENTS OF MEMBER (K) INTO FRAME (K).
      DO51IK=1,MS
      READ33,I,J,K,P,Q,PJ,AR,ANG
   33 FORMAT(3I3,5E10.4)
      ANG=3.1416/180.*ANG
      A(I,I)=A(I,I)&4.0*E*P/Q
      A(I,K)=A(I,K)-6.0*SIN(ANG)*E*P/(Q*Q)
      A(K,I)=A(I,K)
      A(K,K)=A(K,K)&E*AR*(COS(ANG))**2/Q
      A(K,K)=A(K,K)&12.0*E*P*(SIN(ANG))**2/Q**3
   51 A(J,J)=A(J,J)&G*PJ*SIN(ANG)/Q
      DO44IK=1,MC
      READ31,I,J,K,L,MM,NN,P,Q,PJ,IH1,IH2
   31 FORMAT(6I3,3E10.4,2I3)
      P=P*E*20736./Q
      PJ=PJ*20736.
      QQ=Q
      IF(IH1-1)46,47,47
   46 IF(IH2-1)48,49,49
   47 IF(IH2-1)50,44,44
   48 R=4.0
      Q=6.0/Q
      GO TO 52
   49 K=N&1
      GO TO 53
   50 I=N&1
   53 R=3.0
      Q=3.0/Q
   52 IF(I-N)9,9,10
    9 A(I,I)=A(I,I)&R*P
      IF(J-N)11,11,12
   11 A(I,J)=A(I,J)&Q*P
      A(J,I)=A(I,J)
   12 IF(K-N)13,13,14
   13 A(I,K)=A(I,K)&2.0*P
      A(K,I)=A(I,K)
   14 IF(L-N)15,15,16
   15 A(I,L)=A(I,L)-Q*P
      A(L,I)=A(I,L)
   10 IF(L-N)17,17,16
   17 A(L,L)=A(L,L)&Q*Q*P*.33333333
      IF(J-N)18,18,19
   18 A(J,L)=A(J,L)-Q*Q*P*.33333333
      A(L,J)=A(J,L)
   19 IF(K-N)20,20,16
   20 A(K,L)=A(K,L)-Q*P
      A(L,K)=A(K,L)
   16 IF(J-N)21,21,22
   21 A(J,J)=A(J,J)&Q*Q*P*.33333333
      IF(K-N)23,23,8
   23 A(J,K)=A(J,K)&Q*P
```

```
C     PROGRAM ''BRFLAT''    LIST 13.                          PAGE 2
      A(K,J)=A(J,K)
   22 IF(K-N)24,24,8
   24 A(K,K)=A(K,K)&R*P
    8 IF(MM-N)40,40,41
   40 A(MM,MM)=A(MM,MM)&G*PJ/QQ
      IF(NN-N)42,42,44
   42 A(MM,NN)=A(MM,NN)-G*PJ/QQ
      A(NN,MM)=A(MM,NN)
   41 IF(NN-N)43,43,44
   43 A(NN,NN)=A(NN,NN)&G*PJ/QQ
   44 CONTINUE
C     INVERT (K) TO GIVE (F)
      DO2I=1,N
      T=A(I,I)
      A(I,I)=1.0
      DO4J=1,N
    4 A(I,J)=A(I,J)/T
      DO2K=1,N
      IF(K-I)5,2,5
    5 T=A(K,I)
      A(K,I)=0.
      DO6J=1,N
    6 A(K,J)=A(K,J)-T*A(I,J)
    2 CONTINUE
      DO45I=1,N
      DO45J=1,NS,5
   45 PUNCH34,A(I,J),A(I,J&1),A(I,J&2),A(I,J&3),A(I,J&4)
   34 FORMAT(5E14.8)
      STOP 9999
      GO TO 1
      END
```

```
C     PROGRAM ''BRDEF''    LIST 15.                           PAGE 1

      DIMENSION F(48,20),P(16),D(48),S(16,20)
    1 DO2I=1,48
      DO2J=1,16,5
    2 READ34,F(I,J),F(I,J&1),F(I,J&2),F(I,J&3),F(I,J&4)
   34 FORMAT(5E14.8)
      IF(SENSE SWITCH 1)3,4
    4 READ30,D(1),D(2),D(3),D(4),D(5),D(6),D(7),D(8)
      READ30,D(9),D(10),D(11),D(12),D(13),D(14),D(15),D(16)
   30 FORMAT(8E10.4)
      DO6I=1,16
      DO6J=1,16,5
    6 READ34,S(I,J),S(I,J&1),S(I,J&2),S(I,J&3),S(I,J&4)
      DO7I=1,16
      T=0.
      DO8J=1,16
    8 T=T&S(I,J)*D(J)
    7 P(I)=T
      DO5K=17,48
      T=0.
      DO9J=1,16
    9 T=T&F(K,J)*P(J)
    5 D(K)=T
   14 DO10I=1,48,8
   10 PUNCH30,D(I),D(I&1),D(I&2),D(I&3),D(I&4),D(I&5),D(I&6),D(I&7)
      STOP9999
      GO TO 1
    3 READ30,P(1),P(2),P(3),P(4),P(5),P(6),P(7),P(8)
      READ30,P(9),P(10),P(11),P(12),P(13),P(14),P(15),P(16)
      DO11I=1,48
      T=0.
      DO12J=1,16
   12 T=T&F(I,J)*P(J)
   11 D(I)=T
      GO TO 14
      END
```

CHAPTER EIGHT

DISCUSSION AND CONCLUSIONS

The previous chapters have described the simulated elasto-plastic seismic response of several structures which were designed to be built in New Zealand and some of these buildings have now been erected. Generally speaking, these structures performed well under recorded earthquake loading and although the response was very much greater than indicated by code forces, the required member ductilities were comparatively modest and could probably be provided with adequate detailing. It should be remembered that these frames have been carefully designed so that there are no weak members and so that the stiffness of the frame does not taper off too rapidly towards the top.

The computed displacements of multi-mass structures responding to major earthquakes when élasto-plastic action was considered were of the same order as those found when only elastic action was considered. The top storey deflection of all the multi-storey buildings studied was less when plastic action was considered.

For some of the frames the ultimate moments were calculated from details of the beam and column sections and for the others the ultimate moments were assigned by using a load factor on the NZSS 1900 Chapter 8 code loads, thereby checking the suitability of this design method. It was found for these well-proportioned frames that the maximum ductility was required in the member with the highest moment ratio and was up to three times this moment ratio.

The major part of the plastic deformation occurred at the ends of the beams, but a collapse mechanism is not reached until hinges have been formed at the ends of all the beams and at bases of the ground floor columns. The upper floor beams do not yield because the moments caused by the lateral design loads are relatively small compared with those caused by the vertical design loads and so these beams have a relatively high capacity for overload by lateral loads. Occasionally sufficient hinges were formed in the columns to give a sway mechanism and this gave higher ductilities in these columns, but most of the plastic deformation was still occurring in the beams and the hinges were not present in the columns for long enough to have a really significant effect, as they obviously could have if they were a dominant form of energy dissipation. It would appear to be feasible to design multi-mass structures so that the columns remain essentially elastic while the beams yield and provide an adequate way of absorbing energy plastically, without giving total collapse.

The computer programs which were used to carry out comparative static analyses of the regular multi-storey frames dealt with in this thesis were developed in Chapter 2. These programs showed that a computer of the capabilities of the IBM 1620 with 40,000 words of storage could be conveniently used to carry out the elastic static analysis of large multi-storey frames of the order of 20 storeys by six bays. It was originally hoped that time would permit the inclusion of the Livesley $\bar{\phi}$ functions in a dynamic response program to take account of the non-linear effects of axial loads, and while it is unlikely that a seismic resistant frame would buckle due to vertical loading during an earthquake, the lateral deflection may be increased because of a reduction in lateral stiffness particularly after the formation of several plastic hinges.

In Chapter 3 a method of determining the elastic response of a multi-mass structure to a recorded earthquake by numerical integration was described. The same method could be used to determine the response of one-degree-of-freedom systems and it was shown how it could be applied to the determination of response spectra and the multi-mass response using the principle of the independence of the normal modes. The elastic response of a multi-storey frame can be conveniently found using a computer similar to the IBM 1620 if the first few modal equations of motion are integrated but, if the equations of motion of the individual masses are integrated, or if the response spectra are required, a faster machine is desirable.

The method of forming the damping matrix for multi-storey frames from the mass matrix and the lateral stiffness matrix is also described. When this method is used it means that the modal equations of motion will be independent and while this is not necessary for determining the response by numerical integration it is highly desirable so that comparative analyses can be made.

It would seem reasonable that the damping forces in the structure, if they are to be considered as viscous damping forces, are probably well represented by inter-floor damping forces, or symbolically by inter-floor dashpots. If a numerical value of force per unit velocity can be assigned to each dashpot, then the viscous damping matrix can be set up, and from this the fraction of critical damping in each mode may be found. Berg[28] has pointed out that the same system of interfloor damping forces gives a smaller fraction of critical damping in an eight-storey frame than in a four storey frame in the first mode. It should be possible to verify whether or not this behaviour is reproduced in the field. At the present time there is no logical way of calculating the value or range of the damping force per unit velocity to be assigned to a structural or a non-structural element. Further testing, and preferably large vibration testing, may establish the amount of damping to be expected from structural components or between floors. Experimental vibration testing has determined the frequency and fraction of critical damping in each mode. Although this testing does not give the nature or form of the damping matrix, the magnitude of the equivalent viscous damping in the first few modes is known for small vibrations. This does enable the damping matrix to be estimated in a reasonably logical manner although the actual nature and form of the damping forces could differ from those assumed.

In Chapter 4 the development of a digital computer program

"DYNEPRES", which calculates the elasto-plastic response of a regular multi-storey frame to an earthquake record, is described. Some modifications which were required to take account of the effects of shear deformation and joint size were described in Chapter 6 and a computer program "JOINT" was developed with these modifications incorporated. It is not claimed that the program developed in this thesis is the most efficient likely to be produced. In fact, there is room for improvement in several places. It would be desirable to incorporate the improvements in static analysis methods developed in Chapter 2. These were originally developed with this in mind but it was found that in order to fit the program into the limited storage of the IBM 1620 computer a less efficient matrix inversion routine was used and this required many less FORTRAN statements, although it had the advantage that once the inverse of $[K^*]$ was found it could be used repeatedly until the properties of the frame were changed by alteration in the pattern of plastic hinges, whereas with an elimination technique taking advantage of the tri-diagonal stiffness matrix, elimination operations must be carried out at every step, as was done by Clough, Benuska and Wilson[16]. The numerical methods could probably be improved, as this program required a very small step interval. The stiffness matrix becomes less well conditioned as the frame becomes more unstable through the formation of plastic hinges and truncation errors become relatively more important. Another point is that when an incremental stiffness relationship is used there is zero stiffness with regard to an increase in the rotation of a plastic hinge. The moment-rotation relationship has been assumed to be of the elastic-perfectly plastic type for mathematical simplicity. It would be expected that this would be fairly representative of the behaviour of a steel section except that strain hardening may perhaps be significant[29]. It can also be a reasonable approximation to the behaviour of a reinforced concrete section as has been shown by Chan[30]. Further destructive testing to determine the tenacity and ductility of steel and reinforced concrete sections may reveal more realistic moment-rotation relationships which could vary after a few reversing yield excursions.

In Chapter 5 the analysis of two frames using the program "DYNEPRES" was described. The first, a 14 storey steel frame from Building A, was excited by the 1940 El Centro earthquake intensified by 50% and it was found that this very flexible frame could survive this major disturbance with quite modest ductility requirements. The maximum member ductility was only 2.74, but the elastic inter-storey deflection under code loading and the plastic drift which occurs under large earthquakes are significant. The deflections under code loading exceed the code requirements slightly. Although these large deformations are not in themselves dangerous, consideration of damage to finishes, partitions and cladding and also of probable panic by occupants, indicates that it would not be desirable to relax the code limitations on inter-storey deflection or to reduce the design seismic coefficient.

The six storey reinforced concrete frame from Building B was analysed with three different yield moments in an attempt to ascertain a suitable load factor to use on the moments caused by code loading to determine the yield moments which would give satisfactory elasto-plastic behaviour. The yield moments for the members of the frame were also calculated from the reinforcement detailed on the working drawings for construction. The lateral displacements and required member ductilities were quite small for this design and this is reassuring as there is a tendency to use shear walls in a building of this height, particularly public buildings, and this is not economical use of material and has the disadvantage of attracting large amounts of earthquake forces because of the inherent lateral stiffness and yet there is not a good mechanism of plastic deformation.

The analyses of the second and third yield patterns indicated that satisfactory elasto-plastic behaviour could be obtained by using a load factor of 1.25 for the beams and 1.50 for the columns. Yielding was then restricted to the beams except for some at the base of the bottom column. However, this position also had the highest moment ratio of any member. This frame was also analysed with more realistic yield moment values with the program "JOINT". Similar results were obtained except that with the third pattern of yield moments based on a load factor of 1.25 for beams and columns, yielding occurred in most of the columns and this indicates that a larger load factor must be used for the column design.

In Chapter 6 the analyses of two reinforced concrete frames were described. The first, a 13 storey two-bay frame from Building D, has short deep spandrel beams for which the yield moments were calculated from the working drawings and this gave reduced deformations and reasonable ductilities with a maximum of 5.69. This frame was deliberately designed so that all the yielding occurred in the beam members and the analysis verified this. The calculation of the yield moments for the columns in a reinforced concrete frame is not necessarily a straightforward operation as the axial forces are an important parameter. Provided that the column spacing is relatively large and the beams of slender proportions compared with their length, then the axial forces in the columns are not greatly affected by the lateral loading and are mainly dependent on the vertical loading. When the beams are relatively deep compared to their length the axial forces in the columns can be greatly influenced by the lateral loading and tensile forces may even be obtained. The calculation of the yield moment of a column with tensile forces may be done according to the A.C.I. code[26] but this is not backed by very much experimental evidence. It would be possible to simulate the ultimate moment-axial load relationships in the computer program, and calculate the yield moment according to the axial load at the end of each step interval. This procedure was not used because the axial force could be quite accurately assumed to be constant for all the frames, except for the one from Building C for which the yielding of the columns was not a significant effect.

The second reinforced concrete frame was 12 storeys high by two bays wide and the elasto-plastic response was determined assuming that the yield moments had been detailed by using a load factor, on the code lateral and vertical loading, of 1.25 for the beams and 1.50 for the columns. Quite large ductilities were obtained near the bottom of the building, particularly at the base of the bottom columns, where a ductility of 9.48 was required. This was also the position of the highest moment ratio even though the columns were designed with a higher load factor than the beams. This is because the base of the bottom column is not affected very much by vertical loading whereas with the lower beams the vertical loading is more significant. It would appear from the limited studies made so far both here and previously[16] that a reinforced concrete frame with relatively slender members is liable to require higher member ductilities than either a flexible steel structure or a spandrel beam type of frame.

In Chapter 7 the analysis of a large railway bridge was considered and it was shown that designing the deck beam to yield at half the maximum seismic elastic moments gave satisfactory elasto-plastic behaviour. The elasto-plastic displacements were slightly less than the displacements computed assuming elastic behaviour and the member ductilities were of reasonable magnitude with a maximum of 4.67.

At present, the only available design method to take account of the behaviour of framed structures under major earthquakes is to assign a yield moment based on a load factor on the moments under code loading. The columns should be prevented from yielding by using a higher load factor and by ensuring that the limit on the moment which a joint can carry is set by the sum of beam ultimate moments and not by the sum of the column ultimate moments Having done this, there is at the moment no way of estimating the magnitude of the plastic deformations which will occur.

It would appear that a load factor of 1.25 on the code loading used to assign the ultimate moments for the beams and 1.50 used for the columns gives reasonably satisfactory behaviour. There is a tendency for the bases of the bottom columns to require a high ductility but this could be reduced by ensuring that this section does not have the highest moment ratio.

The requirement, mentioned in the M.O.W. code "Design of Public Buildings", that the structure should be designed so that the ultimate strength is at least 1.25 times the code vertical and lateral loads does not in itself ensure a satisfactory elasto-plastic behaviour, particularly for the column members.

The New Zealand code NZSS 1900 Chapter 8[5] ensures satisfactory elastic response to small earthquakes by requiring that structures are designed to resist lateral forces given by a seismic coefficient. The code assumes that buildings will not collapse in major earthquakes if the structural components can deform plastically. This is quite reasonable, because well constructed buildings have performed satisfactorily during past earthquakes even though they have been designed for quite small lateral forces. However, the structural form of buildings has been changing rapidly over the last few decades and not many modern buildings have been well tested by severe earthquakes. Also, the magnitude of the plastic deformation which the structural elements must be capable of providing is not specified by the code, which merely states that "All elements ... shall be designed with consideration for adequate ductility". In the publication "Commentary on N.Z.S.S. 1900 Chapter 8"(5), in paragraph 9.3 it is mentioned that the seismic coefficients were derived from elastic spectral acceleration curves using a "ductility factor" of the order of 4. In paragraph 9.4 it is stated that "Satisfactory earthquake resistant properties are dependent on the use of materials with good ductility characteristics although not necessarily to the extent suggested by factors of the order of 4".

There is a danger that a reader will assume that provided individual components are capable of providing a ductility of up to 4, then satisfactory behaviour will be obtained. There is also a danger of confusion between the term ductility factor as applied to the factor needed to reduce the predicted elastic forces to the magnitude of the code forces, and as it is applied to the ratio of the total deformation (elastic and plastic) to the deformation of yield. It has been shown previously[8] for one degree of freedom systems that the factor needed to reduce the maximum elastic forces to the yield forces is of the same order as the ratio of maximum displacement (elastic and plastic) to the yield displacement. These results cannot be extrapolated to multi-degree of freedom systems and the studies of the structures in this thesis have indicated that the member ductility ratio is always greater than the moment ratio, i.e. the ratio of the maximum elastic moment to the yield moment, in the member with the highest moment ratio, that is the relatively weakest member.

In the future, the designer will be quite at liberty to carry out an elasto-plastic analysis to determine the response to a major earthquake using a computer program similar to that developed in this thesis. This will require a relatively large amount of computer time, say a few hours on a machine comparable to the IBM 360/44 and quite a large amount of storage, compared to any elastic analysis. Whether the designer will choose to do this elasto-plastic analysis if not required to by regulation is doubtful. It would be most desirable to have a simple method of estimating the magnitude of the plastic deformations based on some sort of static analysis. At the moment, it is difficult to see how this could be done with sufficient accuracy to be useful.

Although the magnitude of the plastic deformations is a significant design parameter, the number of plastic hinges formed is not usually sufficient to form a collapse mechanism and so the rotation of the plastic hinges cannot be calculated by the simple virtual work method based on collapse analysis. This means that the rotations of the plastic hinges must be calculated from an elastic analysis modified to take account of plastic action.

The alternative to carrying out a lengthy computer analysis is to assume that the determination of the likely magnitude of the plastic deformations is unnecessary, provided that yielding takes place in such a way that collapse mechanisms are not formed, and that the sections where yielding occurs are capable of providing an adequate ductility, possibly up to 10 or 12, and this can probably be done with adequate detailing.

Further destructive tests of reinforced concrete and welded steel joints are required to verify that current building components have the tenacity to provide adequate ductility under repeated reversing loading.

CONCLUSIONS

1. The response of multi-mass structures to major earthquakes is greater than indicated by code forces.

2. The displacements of multi-mass structures responding to major earthquakes when elasto-plastic action is considered are of the same order as those calculated when only elastic action is considered. The top storey deflection of each of the four multi-

storey buildings studied here was reduced when plastic action was considered.

3. The required member ductility was always greater than the maximum moment ratio, by a factor of up to three.

4. Typical multi-storey reinforced concrete frames must be capable of providing member ductilities of greater magnitude than four.

5. Multi-storey buildings can be designed so that the columns remain elastic while the beams yield, thus providing an adequate way of absorbing energy plastically without giving total collapse.

6. The carefully designed New Zealand buildings studied in this thesis should behave satisfactorily under a major earthquake, providing adequate attention has been paid to detailing.

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